Spring 2017 Graduate Prelim Exam: Quantum Mechanics. 18 Feb

Name:

Please pick two problems to complete and indicate clearly which two on the answer sheet.

1. **Harmonic potential**

Consider two electrons in a harmonic potential $(kx^2/2)$, with some interaction $\hat{V}$ between them. The Hamiltonian for this system is

$$\hat{H} = \frac{\hat{p}_1^2}{2m} + \frac{1}{2}kx_1^2 + \frac{\hat{p}_2^2}{2m} + \frac{1}{2}kx_2^2 + V\left(|x_2 - x_1|\right) \tag{1}$$

a) (5 points) Write down the energies of a single electron in the harmonic potential (no interaction $\hat{V}$ of course)? What is the energy of transitions between consecutive states?

b) (5 points) Rewrite $\hat{H}$ of the two-electron system in terms of center-of-mass and relative coordinates.

c) (10 points) Use this form for $\hat{H}$ to solve as much as you can for the energies of the eigenstates, and show that the transition energy from (a) is also a transition energy for the two-electron system.

d) (5 points) Assume that the two electrons have the same spin. Why couldn't the wavefunction for the spatial part have the form $\psi(x_1, x_2) = \phi(x_1)\chi(x_2)$? What is a correct form using the individual functions ϕ and χ?

2. **Delta-function potential**

Consider a particle in one dimension, with mass m, interacting with a potential $V(x) = -|a|\,\delta(x)$. This potential has exactly one bound state.

a) (3 points) What are the dimensions (*i.e.* in terms of time, energy, length, mass, etc.) of the quantity a?

b) (6 points) Find the wavefunction $\psi(x)$ of the bound state, by finding separate solutions with $E < 0$ for the regions $x < 0$ and $x > 0$ that have appropriate behavior at $x = \pm\infty$, and combining them so the wavefunction is continuous. Properly normalize your answer.

c) (3 points) What is the discontinuity in slope of the wavefunction at $x = 0$?

d) (6 points) Use Schrödinger's equation $\hat{H}\psi = E\psi$ with the given V to evaluate

$$\lim_{\epsilon \to 0} \int_{-\epsilon}^{+\epsilon} \frac{d^2\psi}{dx^2} dx \tag{2}$$

e) (3 points) Equate the result to the discontinuity in $\psi'(x)$ and solve to show the energy is

$$E = -\frac{ma^2}{2\hbar^2} \tag{3}$$

f) (4 points) How can you tell from this calculation that there are no other bound states?

3. **Infinite well plus electric field**

Consider the classic "particle in a box" problem in which an electron is confined in a potential given by

$$V(x) = \begin{cases} 0 & |x| < L/2 \\ \infty & |x| > L/2 \end{cases} \tag{4}$$

a) (5 points) Write down the energies and wavefunctions for bound states of the electron.

b) (10 points) Find the selection rules for electric-dipole radiation, *i.e.* which transitions between states n and m are allowed.

c) (10 points) Consider applying a small electric field with corresponding potential $\Delta V(x) = \mathcal{E}x$. Use perturbation theory to find the first-order change in the energies and wavefunctions of the lowest eigenstates. You may leave your answers in the form of (nonzero) integrals.