

Prelim: Analytic Mechanics
Fall 2013

Saturday September 14th 2013
Choose 2 out of the 3 problems.

Problem 1: (20 points)

A particle of mass m moves on a smooth horizontal table. It is attached to a light, inextensible string that passes through a small hole in the table and carries a second mass m , which hangs vertically.

(a) Suppose that the mass on the table moves in a circular orbit of radius R . What is its angular momentum?

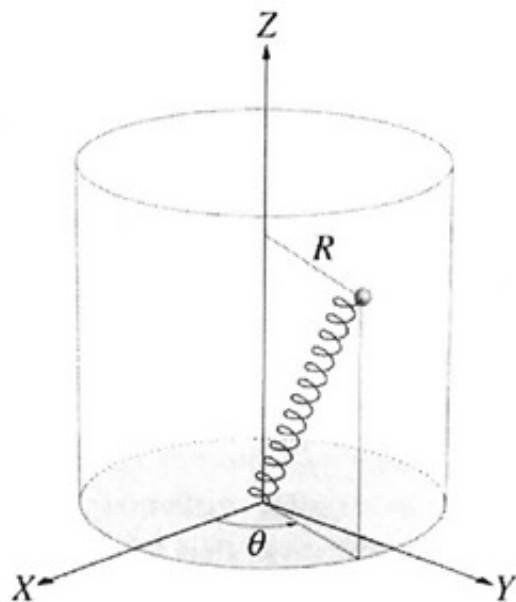
(b) Show that the circular trajectory of radius R is stable and find the frequency ν for small oscillations about it.

Express your final results in both parts a and b in terms of m , R , and g , the acceleration due to gravity.

Problem 2: (20 points)

Imagine a particle of mass, m , confined to an open cylinder of radius, R , and bound to the origin by a spring with spring constant, k , as shown in the figure.

- (a) Write the Lagrangian using cylindrical coordinates
- (b) Derive the Hamiltonian
- (c) Write the equations of motion
- (d) Discuss the equations of motion and their solutions



Problem 3: (20 points)

A particle is moving in the xy-plane on an elliptical path,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

In two seconds it completes three cycles.

- (a) Find its trajectory, $\vec{r}(t) = (x(t), y(t), z(t))$, where $x(t) = a \cos(\omega t)$.
- (b) What force does the particle experience?
- (c) Calculate the angular momentum of the particle. Why must the angular momentum be constant in magnitude and direction?
- (d) Calculate the area ΔS , across which $\vec{r}(t)$ sweeps in one second.