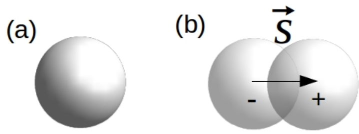
2017 EM Preliminary Exam

1. (a) Please refer to Figure 1 (a) and find the electric field inside a uniformly charged sphere of radius R (charge density ρ).
(b) Please refer to Figure 1 (b). Two spheres, each of radius R and carrying uniform charge $+\rho$ and $-\rho$, respectively, are placed so that they partially overlap. Call the vector from the negative center to the positive center $\vec{s}$. Please show that the electric field in the region of the overlap is constant, and find its value.



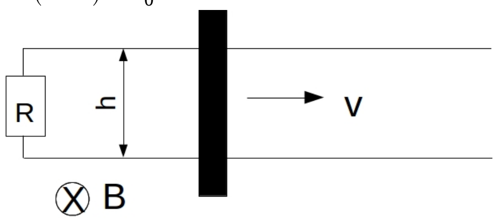
2. Please refer to Figure 2. A metal bar of mass *m* slides frictionlessly on two parallel conducting rails a distance h apart. A resistor R is connected across the rails and a uniform magnetic field B, pointing into the page (as indicated by the tail-of-arrow symbol), fills the entire region (inside and outside the system).

(a) If the bar moves to the right at speed *v*, what is the current in the resistor? In what direction does it flow?

(b) What is the magnetic force on the bar? In what direction?

(c) If the bar starts out with speed v_0 at time t=0, and is left to slide, what is its speed at a later time t ?

(d) The initial kinetic energy of the bar was, of course, $(1/2)mv_0^2$. Where does this energy go? Prove that energy is conserved in this process by show ing that the energy gained elsewhere is exactly $(1/2)mv_0^2$.



3. (a) From the Maxwell equation without any charge or current, show that **E** and **B** satisfy the wave equation.
(b) Show that the solution to the wave equation indeed has the form of a wave. (It is enough to show that a wave satisfies the equation.)
(c) What is the relation between the speed of light, permittivity of free space, and permeability of free space?
(d) By rewriting **E** and **B** in terms of the scalar and vector potentials (V and **A**), show that V and **A** also satisfy the wave equation. (You may use the Lorentz gauge $\nabla \cdot \boldsymbol{A} = \frac{-1}{c^2} \frac{\partial V}{\partial t}$.)