Fall 2018 Physics Preliminary Exam --- Quantum Mechanics

Please pick two problems to complete. Please indicate clearly which two problems you pick.

1. Delta function in the infinite square well

Suppose you put a delta-function bump in the center of the infinite square well:

$$H' = \alpha\delta(x - a/2),$$

where α is a constant.

a) Find the first-order correction to the allowed energies. Explain why the energies are not perturbed for even n.
b) Find the first three nonzero terms in the expansion ($\psi_n^1 =$)of the correction the ground state, ψ_1^1.

Note that $\psi_n^0(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)$.

2. The harmonic oscillator potential

Find the first excited state of the harmonic oscillator by using the appropriate ladder operator

$$a_\pm = \frac{1}{\sqrt{2\hbar m\omega}}(\mp ip + m\omega x)$$

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega}{2\hbar}x^2}$$

3. The energy-time uncertainty principle

Consider the following equation

$$\frac{d}{dt}\langle\hat{Q}\rangle = \frac{i}{\hbar}\langle[\hat{H},\hat{Q}]\rangle + \langle\frac{\partial\hat{Q}}{\partial t}\rangle.$$

a) What does it tell you about the quantity “Q” if $\hat{H}$ and $\hat{Q}$commute?
b) Apply the given equation to the case of $Q = 1$ … comment on your result.
c) Apply the given equation to the case of $Q = H$ … comment on your result.
d) Apply the given equation to the case of $Q = x$ … comment on your result.
e) Apply the given equation to the case of $Q = p$ … comment on your result.