Physics Preliminary Exam Spring 2012
Paper 3 – Electromagnetism
Feb 2nd 2012
3-5pm

Attempt 2 out of the 3 problems

Question 1 (Electromagnetism; 20 points)
A current I flows (in the z-direction) down a long straight wire of radius a. The current I is distributed uniformly across the wire.

a) Find the magnetic field $\mathbf{B}$ everywhere, inside and outside the wire. (Use SI units in this problem.)

b) Now suppose the wire is made of a linear diamagnetic material with susceptibility $\chi_m < 0$. The current I flowing down the wire is now to be viewed as the free current (i.e. the current you control with an applied battery.) Find the H-field inside the wire.

c) Using the result in part b, find the B-field everywhere, expressed in terms of χ_m. Is the result larger, smaller, or the same as the result in part a.

d) Find all the bound surface and volume currents. What is the net bound current I_b flowing down the wire?

Question 2 (Electromagnetism; 20 points)
A series of short questions (5 points each). (Use SI units in this problem.)

a) Find a potential $V(x,y,z)$ for the electric field $\mathbf{E} = ay\hat{\mathbf{x}} + ax\hat{\mathbf{y}} + b\hat{\mathbf{z}}$, where a and b are constants.

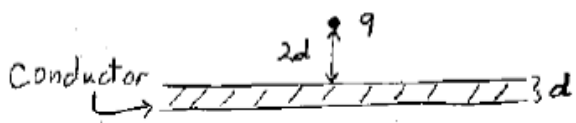
b) Find the volume current $\mathbf{J}$ such that the following electric and magnetic fields satisfy Maxwell's equations

$$\mathbf{E} = \frac{It}{4\pi\epsilon_0 r^2}\hat{\mathbf{r}}, \tag{1}$$

$$\mathbf{B} = 0, \tag{2}$$

where I is constant.

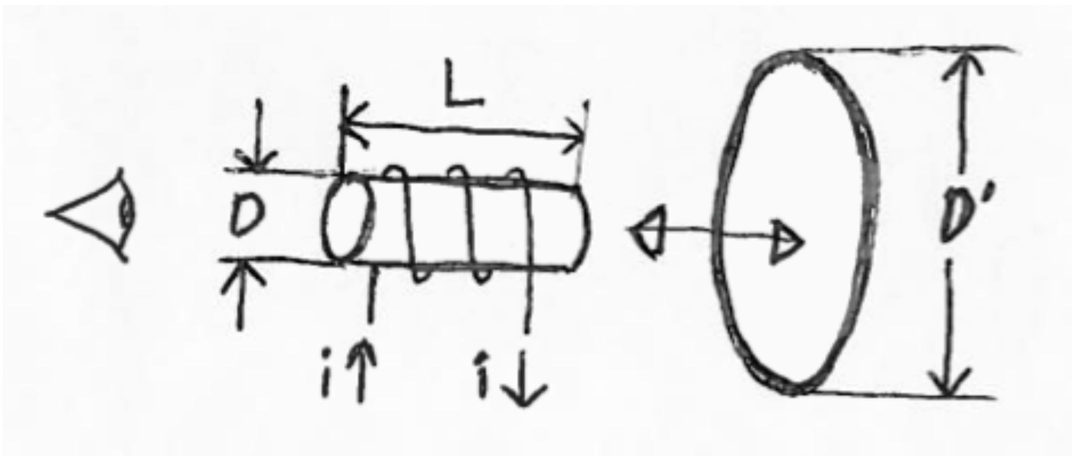
c) A point charge q is placed a distance $2d$ above the surface of an infinite grounded conducting slab of thickness d. (i) What is the magnitude of the force on q? Is it attracted to or repelled from the conductor? (ii) What is the electric field *below* the conducting slab?



d) Two parallel infinite wires carry the same current I propagating in the same direction. Will the force between them be attractive or repulsive? Justify your answer in terms of the Lorentz force law. (You needn't compute the magnitude of the force, just justify the direction of the force.)

Question 3 (Electromagnetism; 20 points)

A solenoid oscillates back and forth, passing completely through a conducting loop with frequency f, as shown in the diagram.



The solenoid has length L and radius R. The solenoid wire has N turns and constant current I flowing clockwise when viewed from the left. The conducting loop has radius R' > R.

(a) What is the average magnitude of the induced emf (voltage) in the loop? (15 pts)

(b) What is the direction of the induced current in the loop when the solenoid starts to pass through the loop from left to right, as viewed from the left? Explain your answer using Lenz's law. (5pts)

Cartesian. $d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}; \quad d\tau = dx\,dy\,dz$

Gradient : $$\nabla t = \frac{\partial t}{\partial x}\hat{\mathbf{x}} + \frac{\partial t}{\partial y}\hat{\mathbf{y}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

Curl : $$\nabla \times \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{\mathbf{z}}$$

Laplacian : $$\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

Spherical. $d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}; \quad d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi$

Gradient : $$\nabla t = \frac{\partial t}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

Curl : $$\nabla \times \mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{\mathbf{r}}$$
$$+ \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial\theta}\right]\hat{\boldsymbol{\phi}}$$

Laplacian : $$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

Cylindrical. $d\mathbf{l} = ds\,\hat{\mathbf{s}} + s\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}; \quad d\tau = s\,ds\,d\phi\,dz$

Gradient : $$\nabla t = \frac{\partial t}{\partial s}\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial t}{\partial\phi}\hat{\boldsymbol{\phi}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial\phi} + \frac{\partial v_z}{\partial z}$$

Curl : $$\nabla \times \mathbf{v} = \left[\frac{1}{s}\frac{\partial v_z}{\partial\phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{\mathbf{s}} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial\phi}\right]\hat{\mathbf{z}}$$

Laplacian : $$\nabla^2 t = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 t}{\partial\phi^2} + \frac{\partial^2 t}{\partial z^2}$$