Physics Graduate Prelim exam

Spring 2009

Instructions:

- This exam has 3 sections: Mechanics, EM and Quantum. There are 3 problems in each section
- You are required to solve 2 from each section.
- Show all work.
- This exam is closed book. No texts of any kind allowed. You are allowed to bring a single sheet of formulae.
- You can use a calculator.

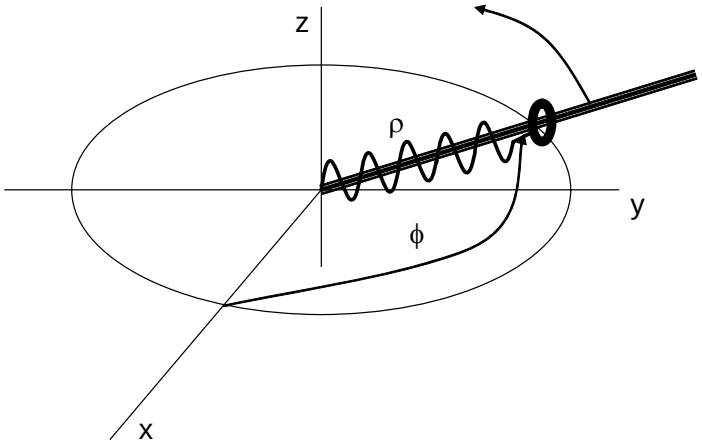
Mechanics
2 out of the 3 problems will be graded

Problem 1

a) Write the equation of motion for a damped harmonic oscillator.
b) Write a general solution for this equation of motion.
c) What is the solution for an under-damped oscillator
d) What is the oscillation frequency
e) Sketch a graph of amplitude vs time for this case

Problem 2

Consider a bead on a frictionless wire that is spinning around an axis perpendicular to the end of the wire. The bead is also connected to the axis of rotation by a spring with relaxed length ℓ and spring constant k. The wire/spring/bead combination spins around with angular frequency ω_s in the x-y plane with cylindrical symmetry (there is no motion in the z-direction).



a) Use two or three sentences to describe what you expect the motion to be like?
b) Using the Lagrangian approach, derive the equation of motion for the bead (DON'T SOLVE IT…JUST WRITE THE EQUATION OF MOTION).

Problem 3

A rigid body consists of three masses fastened as follows: m at (a,0,0), 2m at (0,a,a) and 3m at (0,a,-a)

a) find the inertia tensor
b) find the principal moments and a set of principal orthogonal axes.

Quantum Mechanics
2 out of the 3 problems will be graded

Problem 1

A particle of mass *m* falls under the action of gravity onto a horizontal plane and elastically bounces up. Using Bohr-Sommerfeld quantization, quantize the particle's motion, determine the admissible heights h_n and the energy levels of this system.

Problem 2

Consider an infinite 2 dimensional square box of dimension L x L.
(a) One particle of mass m is confined in this box. Find its wavefunction $\psi(x, y)$
(b) Write down the lowest 3 energy values for this particle. What is the density of states for each of these values? Do this carefully; it matters for the next part.
(c) In the same box, 10 fermions of spin s = ½ are filled. Find the Fermi energy for this system.
(d) Find the total energy of the 10 fermions.

Problem 3

In a quantum state with eigenstates $|\psi_n\rangle$ and eigenenergies E_n for $n \geq 1$, we have $H|\psi_n\rangle = E_n\psi_n$ where the ground state is ψ_1 with energy E_1. Prove that for an arbitrary state $|\psi\rangle$ the expectation value of the Hamiltonian satisfies: $\langle\psi|H|\psi\rangle \geq E_1$.

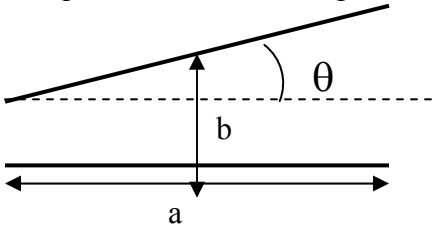
Electricity and Magnetism
2 out of the 3 problems will be graded

Problem 1

Calculate the capacitance of the following two devices:

a. Two coaxial conducting thin cylindrical shells of radii *a* and *b* ($a < b$), have a length $L >> a,b$. The region between the shells is filled with a dielectric material of dielectric constant κ.

b. Two square conducting plates of side *a* are held as shown making a small angle θ with each other. The separation between the plates at the center is *b*.



Problem 2

You are given that the charge density of an electron in the ground state of the hydrogen atom is $\rho(r) = -e/\pi a^3 \, exp(-2r/a)$.

a. Find the electric field and electric potential everywhere due to this electron cloud.

b. How much work is required to place a proton at the center of this cloud?

Problem 3

A particle P of mass M and magnetic dipole moment m is placed on the axis of a circular current loop of radius a and current I (kept fixed) at a distance z_o from the center of the loop. m is aligned in the direction of the loop field.

a. Find an expression for the force of attraction between the particle and the loop.

b. When the particle is released it moves towards the loop, find an expression for the kinetic energy when it reaches the center of the loop.