Physics Graduate Prelim exam

Fall 2008

Instructions:

- This exam has 3 sections: Mechanics, EM and Quantum. There are 3 problems in each section
- You are required to solve 2 from each section.
- Show all work.
- This exam is closed book. No texts of any kind allowed. You are allowed to bring a single sheet of formulae.
- You can use a calculator.

Mechanics

You are required to solve any 2 out of 3

Problem 1

Damped oscillators are described by the following equation:

$$m\ddot{x} + b\dot{x} + kx = 0$$

a) Use diagrams and less than 50 words to explain how one arrives at this equation
b) Show the transformation to the following equation:

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0$$

c) Describe mathematically β and ω_0. What do β and ω_0 physically represent?
d) What three cases of oscillations are generally discussed?
e) Sketch the behavior of each of those three cases as a function of time
f) We often include a term on the right of the equation. What does $f(t)$ represent?

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = f(t)$$

g) Assume that $f(t) = a\cos(\omega t)$ and that β is much less than ω_0. Without going through a huge derivation, how would you expect the oscillator to behave after reaching the steady state for $\omega<\omega_0$? (a graphical description is fine)
h) How would you expect the oscillator to behave after reaching steady state for $\omega>\omega_0$? (again, a graphical description is fine)

Problem 2

The Lagrangian for a mechanical system is $L = a\dot{q}^2 + bq^4$ where q is a generalized coordinate and a and b are constants. The equation of motion for the system is:

a) $\dot{q} = \sqrt{\frac{b}{a}}q^2$

b) $\dot{q} = \frac{2b}{a}q^3$

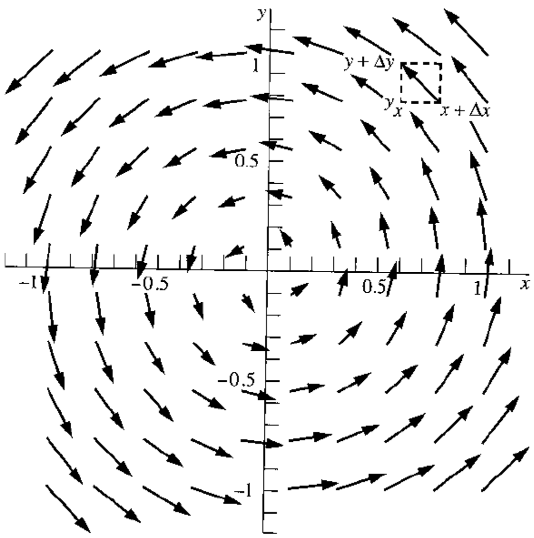
c) $\ddot{q} = -\frac{2b}{a}q^3$

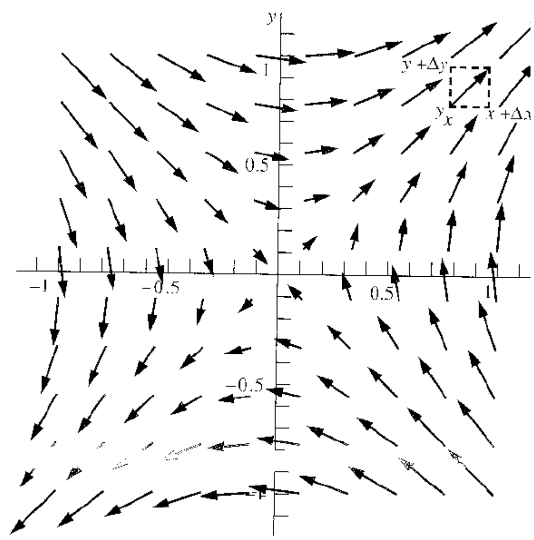
d) $\ddot{q} = +\frac{2b}{a}q^3$

e) $\ddot{q} = \frac{b}{a}q^3$

Note: in order to get full credit you have to show work.

Problem 3





$$\vec{F} = -by\hat{x} + bx\hat{y} \qquad \qquad \vec{F} = by\hat{x} + bx\hat{y}$$

Above are representations of two “force fields” and their corresponding mathematical representations. Arrow points on the graphs represent the magnitude and directions of the force evaluated at the center of each arrow.

a) **Test each of the above forces (either graphically or mathematically) to see if they are conservative.**
b) **Write the potential energy function for the case(s) where the force is conservative.**
c) **Consider the case of a force given by** $\vec{F} = ay\hat{x} + bx\hat{y}$ **where** $a \neq b$ **are both positive numbers. Is this a conservative force?**
d) **Name three things that are special about conservative forces?**

Electromagnetism
You are to solve any 2 out of 3

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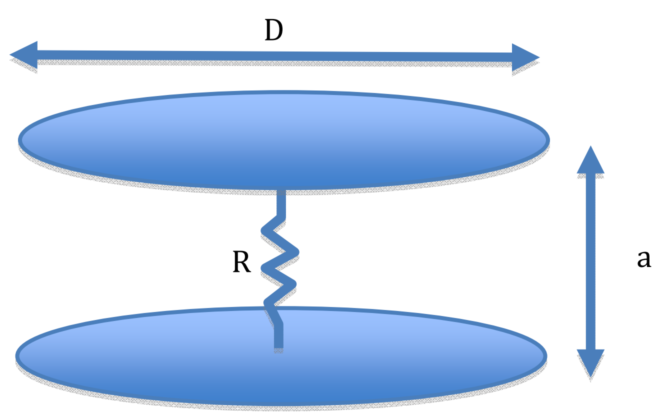
1) A parallel plate capacitor is composed of two metal circular discs of diameter **D**, separated by a distance **a** in vacuum (where D>>a). A resistor of resistance **R** connects the two plates.
   At t=0 the capacitor is suddenly given a charge of $\mathbf{Q_0}$. Neglecting any edge effects write down:

   a) An expression for the capacitance of the capacitor in terms of it's dimensions (2 pts)
   b) An expression for the time dependant charge on the plates (4 pts)
   c) An expression for the time dependant electric field between the plates and indicate its direction on the diagram below. (5 pts)

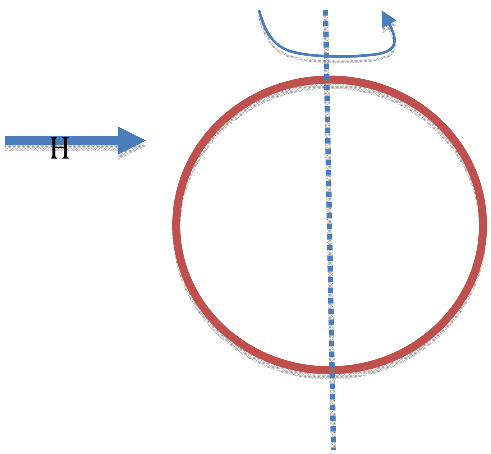


d) Write down three of the following electrodynamics equations defining your constants. Explain their use and context with a diagram. (9 pts)

1) The Biot-Savart Law
2) Ampere's Law
3) Kirchhoff's rule for current junctions
4) Faraday's Law of Induction
5) The Poynting vector

2) Describe the operation of the transformer with a diagram and explain why they are used in long distance power transmission. (5 pts)

A thin copper ring of radius 10cm rotates about an axis perpendicular to a uniform magnetic field H of 0.02T. The rotational axis is a diameter of the ring. (15pts)



If the initial angular frequency of rotation is ω_0 , calculate the time it takes for the frequency to decrease to ω_0/e.

Assume the energy loss goes into Joule heating in the ring.

The conductivity of copper is 5.5 x 10^7 s^{-1}, the density of copper is 8.9g/cm^3

CLUE: $I = \frac{1}{2} Mr^2$ for a thin ring

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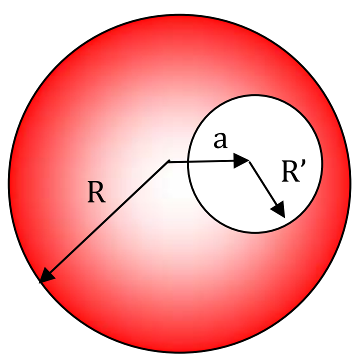
3) A sphere of radius R is uniformly charged with a charge density ρ. Inside it is a spherical cavity of radius R' whose center is at a distance *a* from the center of the sphere. Find the electric field vector (i) inside the cavity, (ii) outside the cavity but inside the sphere and (iii) outside the sphere.



Quantum Mechanics
You are required to solve any 2 out of 3

Quantum harmonic oscillator describes a particle in an harmonic potential with the Hamiltonian $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$. For a quantum particle, the coordinate operator x and the momentum operator p obeys the commutation relation $[x, p] = i\hbar$
Define the following operators

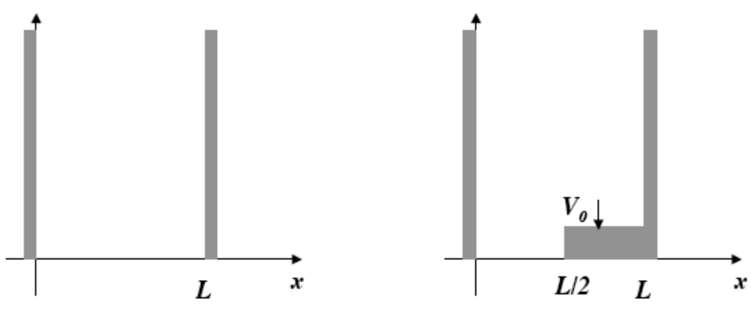
$$a_{\pm} = \frac{1}{\sqrt{2\hbar m\omega}}(\mp ip + m\omega x)$$

1. derive the commutation relation $[a_-, a_+]$
2. express the Hamiltonian H in terms of a_- and a_+
3. if wave function $\psi(x)$ is an eigenstate of the harmonic oscillator Hamitonian with $H\psi(x) = E\psi(x)$, show that $a_+\psi(x)$ is also an eigenstate with eigenenergy $E + \hbar\omega$.
4. The ground state wave function of the oscillator is

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega}{2\hbar}x^2}$$

derive $\langle \Delta x^2 \rangle$ and $\langle \Delta p^2 \rangle$ and show that they satisfy uncertainty principle.

Consider a particle in a one dimensional infinite square well. The length of the well is L.



(a) (b)

1. calculate the first three eigenstates of the infinite square well.
2. with an initial wave function $\psi(x,0) = \psi_1(x) + \frac{1}{2}\psi_2(x)$, write down the time evolution $\psi(x,t)$

3. in (b), a time-independent perturbation is applied with

$$V(x) = \begin{cases} V_0, L/2 < x \le L \\ 0, otherwise \end{cases}$$

use perturbation theory to calculate the new ground state energy, only considering the first three states.

In a double well potential, particle exhibit quantum tunneling between the wells. Here, we assume only one local state is considered in each of the potential well. The states are labeled as $|L\rangle$ and $|R\rangle$, with the energy E_L and E_R respectively. The quantum tunneling rate is given as t_0 and the tunneling Hamiltonian is
$V = t_0|L\rangle\langle R| + t_0|R\rangle\langle L|$

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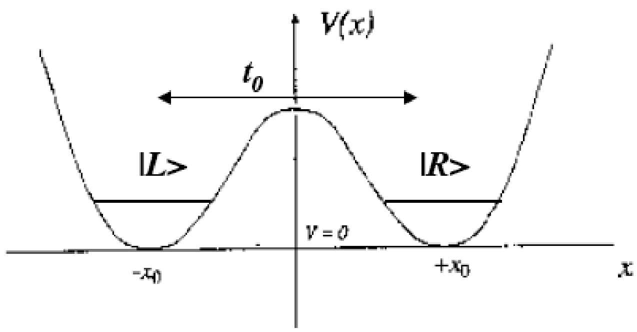

1. for the two level system of $|L\rangle$ and $|R\rangle$ including the tunnel Hamitonian, solve the eignestates and eigenenergies $\lambda_\pm$, (eigenstates can be expressed in terms of $\lambda_\pm$, no need to simplify)

2. for $E_L = E_R$, write down the the energy difference between the new eigenstates. let the initial state be $|L\rangle$. Write down the time evolution of the state.

3. assume $t_0 << | E_L - E_R |$, use perturbation theory to write down the second order modification of the energy E_L.