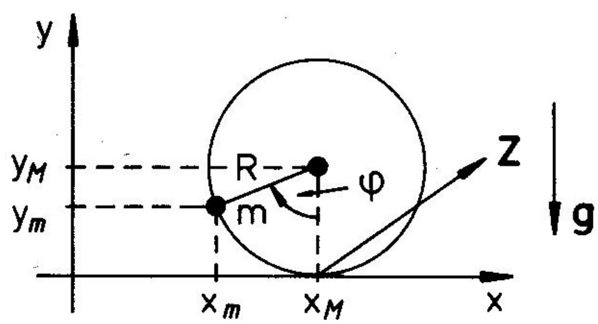
Physics Preliminary Exam Fall 2010
Paper 1 – Classical Mechanics
Sept 14th 2010
6-8pm

Attempt 2 out of the 3 problems

Question 1

A homogeneous circular disk (radius R, mass M) has an additional point-like mass $m=M/2$ located at its rim. The disk rolls without friction, and without gliding on a horizontal line while experiencing gravitation.



1. Calculate the coordinates x_M, y_M of the center of the disk as function of the angle φ. Reference in a way that $\varphi=0$ when $x_M=0$. (1pt)

2. Calculate the coordinates of the point-like mass x_m, y_m, as well as the coordinates, x_{CM} and y_{CM}, of the center of mass of the combined system of disk and point-like mass as a function of φ. (3pts)

3. Calculate the kinetic energy, $T(\varphi, d\varphi/dt)$, and the potential energy $V(\varphi)$.(4pts)

4. Construct the Lagrange function $L(\varphi, d\varphi/dt)$ and the corresponding equation of motion for φ. (4pts)

5. Calculate the constraining force on the disk caused by the horizontal line. (4pts)

6. Because of the additional point-like mass, a large enough initial velocity $v=dx_M/dt$ can cause the disk to "lift off" the horizontal line. How large does v have to be for that to happen when φ=2p/3? (4pts)

Question 2

A clever way of visualizing motion is through the concept of *phase space*. For example, one can visualize the motion of a simple harmonic oscillator by representing position on the *x*-axis and acceleration on the *y*-axis.

1) What does the phase space diagram for a simple harmonic oscillator without damping look like look like? (make a 2D sketch, without scaling)

2) What does the phase space diagram for a very weakly damped harmonic oscillator look like? (make a 2D sketch, without scaling)

3) Write the solution to a simple harmonic oscillator (undamped).

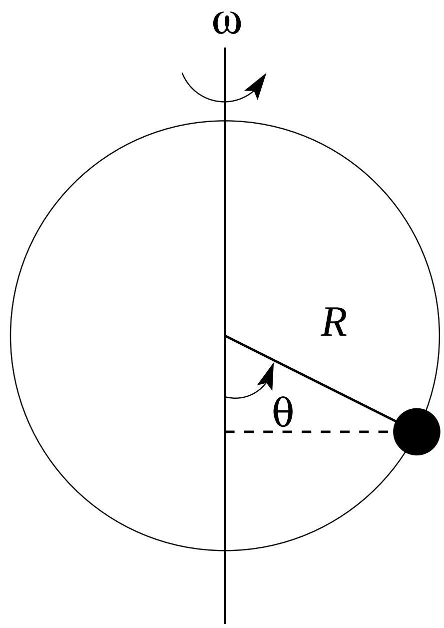
4) What equation describes the undamped oscillating object's trajectory in phase space?

5) Consider a particle of mass *m* subject to a force of strength $+kx$, where x is the displacement of the particle from equilibrium. Calculate the phase space trajectories of the particle.

(20pts)

Question 3 (Classical Mechanics; 20 points)

A circular wire hoop of radius R rotates about a vertical axis passing through its diameter. The hoop rotates with a fixed angular velocity ω. A bead of mass m is threaded on the wire hoop and moves along the hoop without friction. Denote the position of the bead on the hoop by the angle θ, measured with respect to the bottom of the hoop. Denote the acceleration due to gravity by g.



(i) Write down the Lagrangian $L(\theta, \dot{\theta})$ for this system in terms of the generalized coordinate θ.

(ii) Find *all* equilibrium positions θ_0, i.e. those values of θ for which the bead's position relative to the hoop does not change (assuming $\dot{\theta} = 0$ initially). You should find that there is a critical rotation speed ω_c, above which there are more equilibria than below. What is ω_c, expressed in terms of R, m, and g?