# Physics Preliminary Exam Fall 2010
# Paper 3 – Electromagnetism
# Sept 16th 2010
# 6-8pm

## Attempt 2 out of the 3 Questions

### Question 1

A particle with mass, $m$, and charge, $q$, is moving under the influence of a spatially and temporally constant magnetic field, $\boldsymbol{B}=(0,0,B_0)$.

1. What is the particle's equation of motion? **(1pts)**
2. Show that $|\boldsymbol{v}|=|d\boldsymbol{r}/dt|$=const. **(2pts)**
3. Show that the angle between the velocity, $\boldsymbol{v}=d\boldsymbol{r}/dt$ and the field **B** is constant. **(2pts)**
4. For $\mathbf{r}(t=0)=\mathbf{r}_0$ and $\mathbf{v}(t=0)=\mathbf{v}_0$ determine a relation between **r**(t) and **v**(t). **(2pts)**
5. What can be said about $\mathbf{v}_{||}$ (the component parallel to **B**) and the $\mathbf{v}_{\perp}$ (the component perpendicular to **B**)? **(2pts)**
6. f(t) is the angle between $\mathbf{v}_{\perp}$ and the $\mathbf{e}_1$-axis. Show: f(t)=-wt+a; w=qB/m; a=const. **(1pt)**
7. Now chose $\mathbf{e}_2||\mathbf{v}_{0\perp}=(\mathbf{e}_3\times(\mathbf{v}_0\times\mathbf{e}_3))$, which implies $\mathbf{e}_1||(\mathbf{v}_0\times\mathbf{e}_3)$. Determine the full solution to **v**(t) and **r**(t). **(3pts)**
8. Under which conditions does the particle move on a circular orbit perpendicular to **B**? **(1pt)**
9. Of what geometrical form is the general solution? **(1pt)**
10. Now let $\boldsymbol{B}=(0,0,B_0)$, $\boldsymbol{r}_0=(0,0,0)$, $\boldsymbol{v}(t=0)= v_0 3^{-1/2}(1,1,1)$. How does the trajectory of the charged particle change if an additional electric field, $\boldsymbol{E}$, is applied, and which of the following entities remain constant, $v_x$, $v_y$, $v_z$, $v_{\perp}$, $|\mathbf{v}|$, **p**, $|\mathbf{p}|$, $E_{kin}$?
    a. $\mathbf{E}=E_0(0,0,1)$ **(2pts)**
    b. $\mathbf{E}=E_0(1,0,0)$ **(3pts)**

**Question 2 (20pts)**

Consider a model of the hydrogen atom with the proton being a point charge $+e$ situated at $r=0$, surrounded by a spherically symmetric electron cloud of total charge $-e$. You are given that the charge density of the electron cloud in the ground state of the hydrogen atom is $\rho(r) = -e/\pi a^3 \exp(-2r/a)$.

The atom is placed in a uniform electric field $E$. Assuming the electron cloud remains spherically symmetric,

(a) compute the induced dipole moment $p$.

(b) Show that $p=\alpha E$ and find the polarizability $\alpha$ explicitly.

**Question 3 (20pts)**

To measure the magnetic susceptibility of a sample, a physicist constructs an LC circuit consisting of a capacitor of capacitance $C$ connected across a long solenoid of length $l$, cross-sectional area $A$ with $N$ turns of wire.

(a) What is the resonance frequency, $f_0$, of this LC circuit?

(b) When a sample is placed in the solenoid, the measured resonance frequency drops to $f_s$. What is the magnetic susceptibility of the sample?