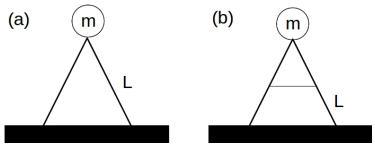
Spring 2017 Physics Preliminary Exam: Classical Mechanics

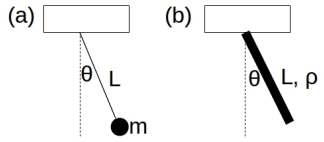
Please pick two problems to complete and clearly indicate which two problems you choose.

1. Consider a simple ladder composed of two identical wood planks with uniform mass density ρ per unit length and length L. The two planks touch each other at the top vertex, but the vertex does not generate additional force. The ladder is place on a smooth surface with no friction. The gravitational acceleration is g. Please refer to Figure 1 for the setup.



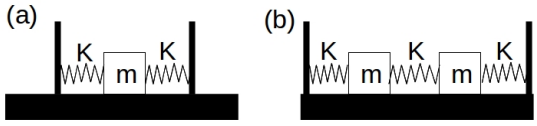
(a) If a point mass m is place on the top of the ladder, quantitatively show that the system is unstable in general. Please also find out the special case when the system can be stable.
(b) By adding a massless (but very strong) strap horizontally in the middle (at L/2 on each lag), please show that the system can be stabilized.

2. Please refer to Figure 2. The rectangles represent a fixed ceiling and the objects are hung by massless and frictionless pivots. The gravitational acceleration is g.



(a) A pendulum is composed by a massless string of length L and a point mass m attached to its lower end. (1) Please write down the Lagrangian of the system. (2) Show that the Lagrangian-Euler equation leads to the equation of motion. (3) Assuming small-angle oscillations, simplify the equation of motion, and find the oscillation frequency.
(b) A pendulum is composed of a thin wood stick of uniform mass density ρ per unit length and length L. (1) Please find the moment of inertial of the stick around its pivot on the top. (2) Write down the Lagrangian of the system. (3) Show that the Lagrangian-Euler equation leads to the equation of motion. (4) Assuming small-angle oscillations, simplify the equation of motion, and find the oscillation frequency.

3. Please refer to Figure 3. Two systems consisting of point masses (with mass m) and springs (with spring constant K) are placed on smooth surfaces without any friction. The ends of the system are fixed (by the vertical rods).



(a) For the system with one mass and two springs: (1) Write down its Lagrangian. (2) Derive the equation of motion from Lagrangian-Euler equation. (3) Find the normal-mode frequency of the oscillation.
(b) For the system with two masses and three springs: (1) Write down its Lagrangian. (2) Derive the equation of motion from the Lagrangian-Euler equation. (3) Find the normal-mode frequencies of the oscillation.