2017 Electromagnetism Preliminary Exam

Please pick two problems to complete. Please clearly indicate which two problems you choose.

1. Electromagnetic waves

Use Maxwell's equations for free space to arrive at the following wave equations

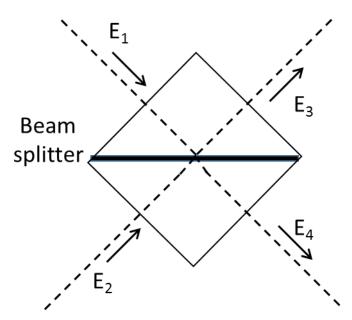
$$\nabla^2 \vec{E} = \varepsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^{2}\vec{E} = \varepsilon_{0}\mu_{0}\frac{\partial^{2}\vec{E}}{\partial t^{2}}$$
$$\nabla^{2}\vec{B} = \varepsilon_{0}\mu_{0}\frac{\partial^{2}\vec{B}}{\partial t^{2}}$$

2. Short answer questions about optics

- a) What are the SI units for the amplitude of the electric field?
- b) What are the SI units for the wavelength of an electromagnetic wave?
- c) What are the SI units for the frequency of an electromagnetic wave?
- d) What are the SI units for the intensity of an electromagnetic wave?
- e) What are the SI units for the power of an electromagnetic wave?
- f) Sketch a harmonic electromagnetic wave and label the wavelength and amplitude on that sketch?
- g) What is the mathematical relationship between intensity and electric field amplitude for an electromagnetic wave?
- h) What is the mathematical relationship between the intensity and power for an electromagnetic wave?

3. The lossless optical beam splitter



The core of many interferometric devices is a beam splitter.

Figure 1: The lossless beam splitter. We assume fields are input at port 1 and port 2, and the fields emerging from port 3 and port 4 are determined by complex reflection and transmission coefficients.

(a) Show that there is a $\pi/2$ phase difference between the reflected and transmitted electromagnetic fields when interacting with a lossless beam splitter.

(b) Sketch a plot of the variation of refractive index as a function of frequency for a typical dielectric material. Label the most important features on that plot.

(c) Use a few equations and perhaps sentence or two to explain how interference gives us information about the phases of the interfering beams.