2018 Physics Graduate Preliminary Exam Electromagnetism Please pick two problems and indicate on your answer sheet which two you choose.

1. Suppose we place a point charge q at the center of a neutral spherical conducting shell. (See the figure below. The shaded region contains the conducting material.) The inner and outer radius is a and b, respectively.

(a) It will attract charge to the inner surface and the outer surface of the conductor. How much induced charge will accumulate there? (Please show your answers for the inner and outer surfaces separately.) (b) Find *E* and V as functions of r in the three regions r < a, a < r < b, and r > b.



2. A steady current *I* flows down a long cylindrical wire of radius R. (See the figure below.) Find the magnetic field, both inside and outside the wire, if

(a) The current is uniformly distributed over the outside surface of the wire;

(b) The current is uniformly distributed over the whole wire.



3. (a) Show that the relations between the fields **E**, **B** and the potentials V, **A** are invariant under a gauge transformation $A' = A + \nabla \lambda$, $V' = V - \partial \lambda / \partial t$.

(b) Show that if we choose the Lorentz gauge $\nabla \cdot A = \frac{-1}{c^2} \frac{\partial V}{\partial t}$, the potentials V and A both satisfy wave equations in vacuum (without any charge or current).