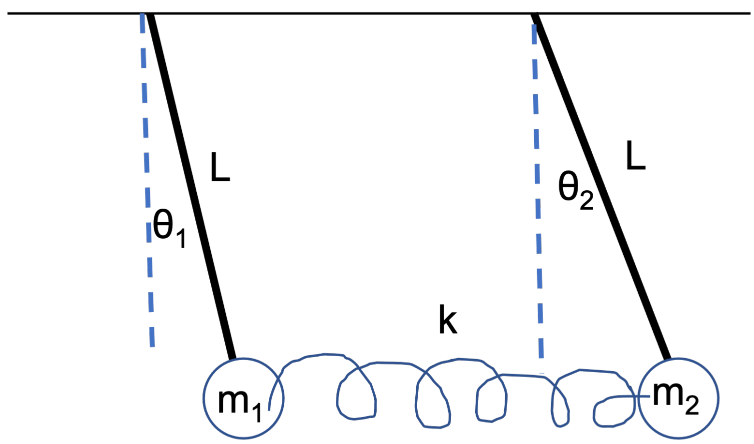
Do **2 out of the three** problems. Put a star next to the ones you want to count for the exam. Partial credit will be given for writing down appropriate equations and outlining the steps you would have used.

1. (15 points) Two pendulums with point masses m_1 and m_2 and identical lengths L are connected by a massless spring with spring constant k. The angle between the first pendulum and vertical is labelled θ_1 and the angle between the second pendulum and vertical is θ_2.

(a) Write down the Lagrangian for this system in the limit of small oscillations (θ_1 and $\theta_2 \ll 1$). In this limit, you may assume that the spring only stretches in the x direction. The small angle approximations to second order are $\sin\theta \sim \theta$, $\cos\theta \sim 1 - \theta^2/2$)

(b) In the limit of small oscillations (θ_1 and $\theta_2 \ll 1$), find the normal modes and corresponding eigenfrequencies of oscillation. In addition, describe the normal modes with words and/or pictures.



2. (15 points) A particle with mass m moves in a central force described by:

$$F(r) = -\frac{k}{4}r^{-4} \tag{1}$$

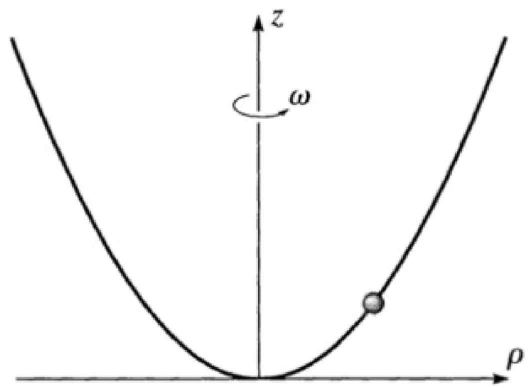
where k is greater than zero

(a) What is the Lagrangian for this system in polar coordinates? (You do not need to re-derive the kinetic energy in polar coordinates if you already know it. Also you may pick your coordinate system such that the particle's motion is in a plane).

(b) What are the Euler-Lagrange equations of motion? Show that the angular momentum, ℓ of the particle is conserved.

(c) Write down the total energy of the particle in terms of the angular momentum ℓ, and r. What is the radius r_o of a circular orbit, in terms of the total energy E and the angular momentum ℓ?

2. (15 points) A particle with mass m is constrained to slide on a frictionless wire which itself is rotating about the vertical axis with angular frequency ω. The wire has a fixed shape of $z = k/4r^4$, where k is positive.



(a) Write down the Lagrangian in terms of the generalized coordinate r. (You do not need to re-derive the kinetic energy in cylindrical coordinates if you already know it).

(b) Find the equations of motion of the particle, and identify any equilibrium positions for the particle.

(c) How would you determine whether the equilibrium positions were stable? (you can use words here, do NOT actually solve).