Jan. 2022 EM Prelim

UC Merced

Complete 2 of the 3 following problems.

1. Consider a sphere of charge with radius R which is filled with a constant charge density ρ.
   (a) What is the electric field, $\vec{\mathbf{E}}$, and potential, V, everywhere? (Make sure $V \to 0$ at infinity.)
   (b) Compute the total energy required to assemble the charge using only $\vec{\mathbf{E}}$.
   (c) Compute the total energy required to assemble the charge using only ρ and V.
   (d) Suppose we keep the **total** charge constant, but move it all to the surface of the sphere. What is the resulting **surface charge density**, σ?
   (e) If we moved the charge to the surface, would the electric field change inside the sphere, outside the sphere, both, or neither? (Also, explain how you came to this answer.)
   (f) If we move the charge to the surface does the stored electromagnetic energy increase, decrease, or stay the same? (Also, explain how you came to this answer.)

2. Suppose we have an oscillating planar current which produces an electromagnetic wave with the following $\vec{\mathbf{B}}$ field:

$$\vec{\mathbf{B}}(z > 0) = B_0 \cos(kz - \omega t)\hat{\mathbf{x}} \tag{1}$$

$$\vec{\mathbf{B}}(z < 0) = -B_0 \cos(-kz - \omega t)\hat{\mathbf{x}} \tag{2}$$

   There is no free charge anywhere in the problem, and no current anywhere except on the plane (defined by $z = 0$).
   (a) Using one of Maxwell's equations, find $\vec{\mathbf{E}}$ above and below the plane.
   (b) Using Maxwell's equations, prove that this wave must be moving at the speed of light.
   (c) What is the time varying current density, $\vec{\mathbf{K}}$, on the plane? Be sure to include the direction as well as the magnitude!
   (d) What is the flux of electromagnetic momentum, $\vec{\mathbf{S}}$, above and below the plane? You should be able to eliminate k and ω from your final answer.
   (e) Show that the $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ field store equal amounts of energy.

3. Suppose we have a flexible circular loop of wire with radius a, laying flat in the $z = 0$ plane in an external magnetic field. The wire has resistance R.

   We will consider four cases:

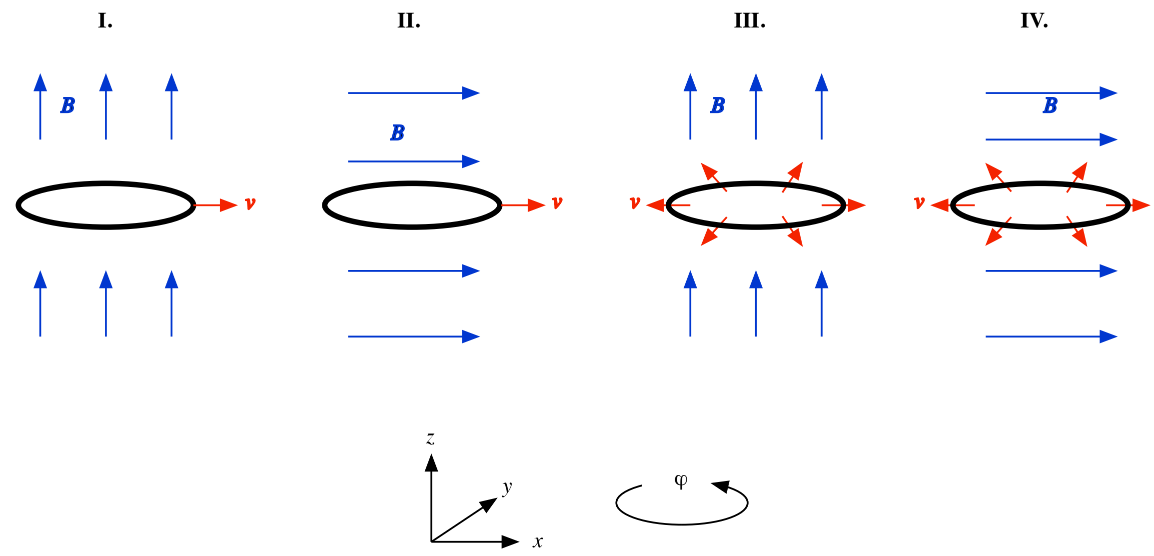
   I. Magnetic field: $\vec{\mathbf{B}} = B_0\hat{\mathbf{z}}$, wire loop moving at a speed $\vec{\mathbf{v}} = v_0\hat{\mathbf{x}}$.

II. Magnetic field: $\vec{\mathbf{B}} = B_0\hat{\mathbf{x}}$, wire loop moving at a speed $\vec{\mathbf{v}} = v_0\hat{\mathbf{x}}$.

III. Magnetic field: $\vec{\mathbf{B}} = B_0\hat{\mathbf{z}}$, with the center of the loop stationary but the radius increasing so that $a(t) = a_0 + vt$.

IV. Magnetic field: $\vec{\mathbf{B}} = B_0\hat{\mathbf{x}}$, with the center of the loop stationary but the radius increasing so that $a(t) = a_0 + vt$.

You may assume v is a constant.



(a) Exactly one of these cases will induce a non-zero electromotive force (EMF) in the wire. Explain which and why.

(b) For the case with non-zero EMF: what is the current, I, flowing in the wire? Make sure the sign is correct: positive values should correspond to current flowing in the $+\hat{\phi}$ direction, using a standard cylindrical coordinate system. (Note: you may assume that R is large, so that the current is small and the additional EMF created by the self-inductance of the loop is negligible.)

(c) For the case with non-zero EMF: what is the rate at which energy is dissipated by moving/stretching the wire?

(d) For the case with non-zero EMF: As the loop moves/stretches and current flows, it will produce an additional magnetic field. In which direction does this extra magnetic field point at the center of the loop? Explain how you know this.

(e) For the case with non-zero EMF: Suppose the velocity is not constant, but rather the loops is initially stationary and then starts suddenly moving (i.e. $v(t < 0) = 0$ and $v(t > 0) = v_0$). As a result, the induced current will change, and there will be an additional EMF due to the self-inductance of the wire at $t = 0$. Does this extra EMF have the same or opposite sign as the EMF you previously identified? Explain how you know this.