Prelim: Analytic Mechanics
Spring 2016

Choose 2 out of the 3 problems.

Problem 1: (*20 points*)
Three identical objects, A, B, C each of mass of m, are sitting in a straight line, in that order, connected by springs of spring constant k. There is one spring connecting A and B and one spring connecting B and C. The motion is confined to one dimension. At $t = 0$, the masses are at rest at their equilibrium positions. Mass A is subjected to a force of $F = fcos(\omega t)$, $t > 0$. Calculate the motion of mass C. All surfaces are frictionless.

Problem 2: (*20 points*)

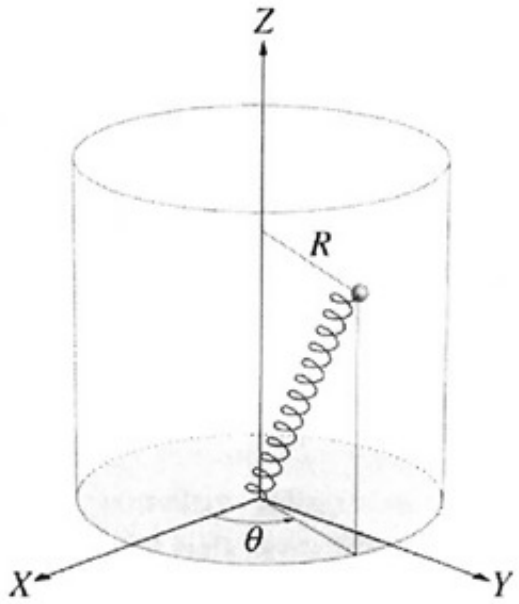
Imagine a particle of mass, m, confined to an open cylinder of radius, R, and bound to the origin by a spring with spring constant, k , as shown in the figure.

(a) Write the Lagrangian using cylindrical coordinates

(b) Derive the Hamiltonian

(c) Write the equations of motion

(d) Discuss the equations of motion and their solutions



Problem 3: (*20 points*)

An empty barrel of mass m and volume $V = \Pi R^2 H$ (cylinder of radius R and height H) is floating in an ideal fluid of constant density ρ. The sides and bottom of the barrel are of negligible thickness and the top is open. At $t = 0$ a very small hole of area a is created in the bottom of the barrel, so that it begins to fill up and eventually sinks. The leak rate is slow enough that the barrel remains in static equilibrium (its velocity and acceleration are negligible compared with all other scales in the problem).
a) What is the difference in the fluid levels Δh inside and outside the barrel? Show that this difference is constant until the barrel sinks.
b) What is the difference ΔP in the fluid pressure across the bottom of the barrel, ie. between the inside and outside surfaces of the barrel floor?
c) What is the velocity of the fluid in the hole? Hint: Consider the net momentum per unit time coming through the hole and set that equal to the net force. The flow is steady-state.
d) How much time elapses between the instant the leak appears and the time the barrel is submerged?