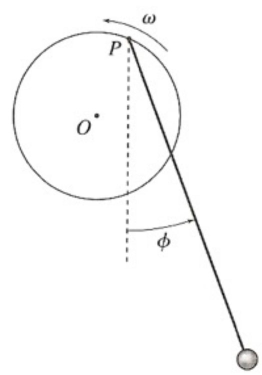
 Name: ____________________________

Prelim
September 2016

Please choose two (2) problems to complete. Please indicate here which two problems you choose:

1. A simple pendulum (mass m, length l) has its point of support attached to the edge of a wheel (radius R) that is forced to rotate at a fixed angular velocity ω.
   (a) Write down the Lagrangian.
   (b) Find the equation of motion for the angle of the pendulum ϕ.
   (c) Determine the angular velocity ω_0 of the wheel in resonance with the pendulum in the limit of small amplitude $\phi \ll 1$.
   (d) Describe the motion of the pendulum with the wheel rotating at an angular velocity higher than the above resonant one. How will such motion affected by any external damping due to, for instance, the air drag.



(Use for additional work)

2. Two particles with equal mass m interact via a potential energy $U = \frac{1}{2}kr^2$, where r is the separation between them.
   a Find the equilibrium separation r_o, the distance at which the two particles can circle each other given the angular momentum l.
   b If the particles are disturbed a little from their separation r_o, explain why they will oscillate about the circular orbit.
   c Find the frequency of such small oscillations [Hint: Expand the potential near the equilibrium point in Taylor series and keep up to the 2nd order. $U_{\text{eff}} \approx U_{\text{eff}}(r_o) + U'_{\text{eff}}(r_o)(r - r_o) + \frac{1}{2}U''_{\text{eff}}(r_o)(r - r_o)^2$.]

(Use for additional work)

3. Electrical force is given by $\vec{F} = q\vec{E}$, where $\vec{E}$ is the electric field. Electric force is conservative in "electrostatic" situations (but is *not* conservative in all situations!)
   a Consider a laboratory setup in which a charge q sits in a field $\vec{E} = (z^2 + 1)\hat{k}$. Prove that the resulting force is conservative
   b Deduce a formula for the potential energy as a function of position of this charge
   c You release the charge (it has mass m) at the origin, starting from rest. Describe its motion qualitatively. How fast is it going when it reaches a distance h from the starting point? If the object was in the shape of a bead (*same* mass m and charge q) threaded on a curved, frictionless, rigid wire which started at the origin $(0, 0, 0)$ and ended at some point (x_0, y_0, h) would it also have that same speed you just calculated, or not? (Why?)

(Use for additional work)