Final Exam

Classical Mechanics Preliminary Exam

February 6th 2021

Attempt 2 out of 3

Problem 1: Sliding Pendulum (8+6+6=20 pts)
Consider a mass, m_2, which is attached to a rigid and massless string of length, l. This pendulum swings in a plane under the influence of a homogeneous gravitational field. The suspension point is formed by a mass, m_1, which can slide freely (frictionless) along a horizontal rail.
a) Formulate the Lagrangian and derive the equations of motion.
b) Which conservation laws exist?
c) Solve the equations of motion for both masses. Describe their paths in words.

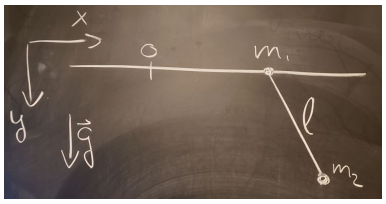

Figure 1: Coupled oscillators

Problem 2: Billiards (6+10+4 pts)
Consider the elastic scattering of two solid spheres of masses m_1 and m_2 and same radius A. Sphere number 2 is at rest in the laboratory frame with its center on the x axis. Prior to the impact on sphere number 2, sphere number 1 moves with constant momentum $\overrightarrow{p}_1 = p_1 \overrightarrow{e}_x$ ($p_1 > 0$), where the path of the center of sphere 1 is parallel to the x axis with a distance of A.
a) Sketch the two spheres in the moment they touch each other. Use your figure to indicate all physical quantities relevant to your calculation. (Hint: Decompose the momenta into convenient components)
b) What are the momenta $\overrightarrow{p}'_1$, $\overrightarrow{p}'_2$ after the impact in the laboratory frame, assuming there is no friction during the scattering event between both spheres?
c) Discuss the special case $m_1 = m_2$.

Problem 3: Surface optimization (20pts)
Find the connection between the two points (x_1, y_1) and (x_2, y_2), which under rotation about the y axis yields the smallest area (see figure).

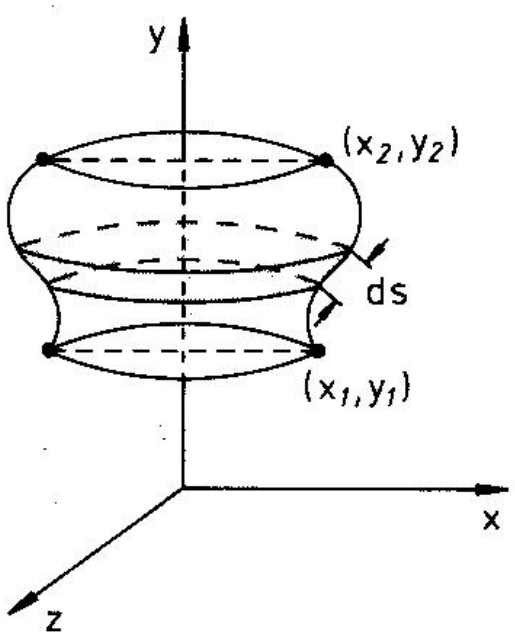

Figure 2: Minimal Rotational Surface