Prelim Exam
Classical Mechanics
Fall 2014

Complete two of the following three questions.

Question 1 (20 points)

An electron (charge e, mass m) is moving in a homogeneous, damped oscillating electric field, $\overrightarrow{E}(t) = \overrightarrow{E}_0(t)\exp(-\alpha t)\cos(\omega t)$, with $\alpha > 0$.

1. Calculate the trajectory $\overrightarrow{r}(t)$. Let the initial conditions be $\overrightarrow{r}_0 \equiv \overrightarrow{r}(0)$ and $\overrightarrow{v}_0 \equiv \dot{\overrightarrow{r}}(0)$. (*10 points*)

2. Determine the initial conditions, $\overrightarrow{r}_0$ and $\overrightarrow{v}_0$, for which $|\overrightarrow{r}(t)|$ remains finite for all $t \geq 0$. Justify your answer. (*10 points*)

Question 2 (20 points)

According to Yukawa's theory of nuclear forces, the attractive force between two nucleons has a potential of the form

$$V(r) = \frac{Ke^{-\alpha r}}{r}, \quad K < 0, \quad \alpha > 0. \tag{1}$$

(a) Find the force.

(b) Find the angular momentum L and total energy E for motion on a circle of radius R.

(c) Find the period of circular motion.

(d) Find the period of small radial oscillations about the circular trajectory.

Question 3 (20 points)

A double pendulum, both parts of which are of equal mass and length, vibrates in a vertical plane. Find the Lagrangian of the system and obtain the equations of motion fo the case where the oscillations are assumed to be small. Find the normal frequencies and normal modes.