E&M Prelim

2015

Please choose 2 (and only 2) problems to answer. On your answer sheet please clearly state which two problems you choose to answer.

For reference, here are the basic equations of electromagnetism in SI units:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \tag{1}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial_t} \tag{2}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{3}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial_t} \tag{4}$$

$$U = \frac{1}{2} \int dV \left[\epsilon_0 |\mathbf{E}|^2 + \frac{1}{\mu_0} |\mathbf{B}|^2 \right] \tag{5}$$

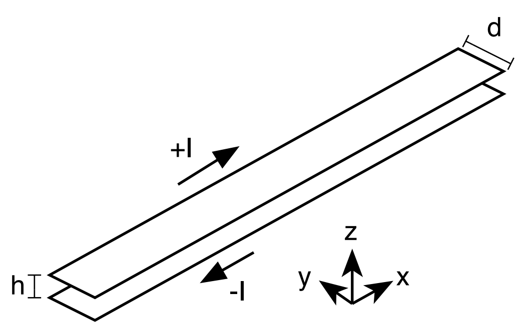
1. (20 pts.) Imagine a model of the hydrogen atom where the nucleus is represented by a point charge, $+q$, and the electron as represented by a sphere of uniform charge density with radius R and total charge $-q$, centered about the nucleus.

   (a) (5 pts.) What is the total electric field in *and* around the 'atom'?

   (b) (10 pts.) What is the difference between the electromagnetic energy of the 'atom' and the two charge distributions separated, i.e. when the nucleus point charge is moved infinitely far away from the electron? Assume the electron spreads out to an infinitely large sphere when it is far from the nucleus, but that the nucleus remains a point. (Hint: make use of the fact that $|\mathbf{E}_2 + \mathbf{E}_2|^2 = |\mathbf{E}_1|^2 + |\mathbf{E}_2|^2 + 2\mathbf{E}_1 \cdot \mathbf{E}_2$)

   (c) (5 pts.) Is the energy of the separated charge more or less than the 'atom'? Does this make sense?

2. (20 pts.) Consider a transmission line consisting of two thin strips with width a, separated by a distance h. You may assume that $a \gg h$ and that the length of the transmission strip is effectively infinite, $\ell \gg a, h$. (Thus you may ignore any 'fringing fields', and treat the field in the gap as constant.) Assume the vector from the bottom to the top strip is pointing in the $+\hat{\mathbf{z}}$ direction, and the direction along the width of the string is $\hat{\mathbf{y}}$. *Be sure to specify the direction of all fields!*



(a) (4 pts.) If we put a linear charge density of $+\lambda$ on the top strip and $-\lambda$ on the bottom strip, what is the electric field in the gap?

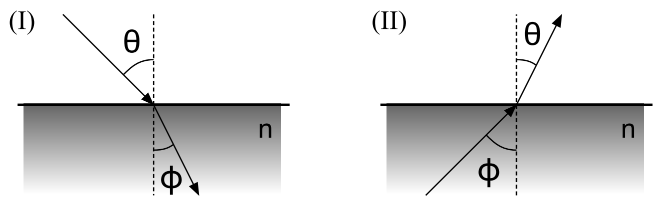
(b) (4 pts.) What is the capacitance per unit length, $C_\ell = C/\ell$?

(c) (4 pts.) If we flow a charge of $+I$ through the top strip (flowing in the $+\hat{\mathbf{x}}$ direction) and $-I$ through the bottom strip, what is the magnetic field in the gap?

(d) (4 pts.) What is the inductance per unit length, $L_\ell = L/\ell$? (Note: I mean the total inductance of the line, not the mutual or self-inductance of either strip. In this case, the total inductance is equivalent to the self-inductance if you imagine one end of the top strip attached to the bottom, so the current is flowing out along top and back in along the bottom.)

(e) (4 pts.) What is the product of the capacitance and inductance per unit length, $C_\ell \cdot L_\ell$? Does this depend on the dimensions of the strip? Is it related to any physical constants?

3. (20 pts.) Consider two cases: (I) a light wave directed at the surface of the material, and (II) a light wave shining out from the inside of a material. Assume vacuum outside the material ($n_{outside} = 1$), and a constant index of refraction, n, inside the material. Represent the angle of incidence outside the material as θ and inside the material as ϕ.



(a) (5 pts.) There is a range of angles for which the surface is totally reflecting, depending on the direction of the light. Using Snell's law, derive the critical angle above/below which this happens.

(b) (5 pts.) Assuming a normal material ($n > 1$), under which conditions is the light totally reflected? More specifically, is it for light starting inside and/or outside the material? Is it totally reflected when the angle of incidence is less than or greater than that derived above? How do we see this from Snell's law?

(c) (2 pts.) Does any of the above depend on polarization?

(d) (4 pts.) Would any of the above change if the index of refraction were negative, $n < -1$?

(e) (4 pts.) Would any of the above change if the index of refraction were positive, but less than one, $0 < n < 1$?