Prelim
September 2017

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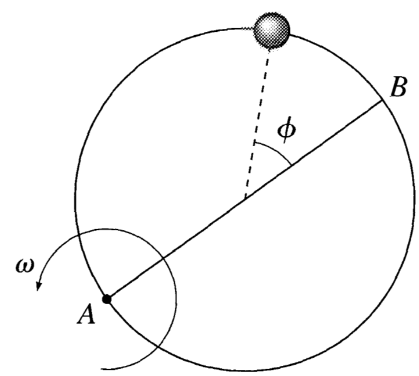
1. Consider a uniform solid disk of mass M and radius R, rolling without slipping down an incline which is at angle θ to the horizontal. The instantaneous point of contact between the disk and the incline is called P.
   (a) Draw a free-body diagram, showing all forces on the disk.
   (b) Find the linear acceleration $\dot{v}$ of the disk by applying the angular momentum equation $\dot{\mathbf{L}} = \mathbf{\Gamma}^{\text{ext}}$ for rotation about P. (Remember that the moment of inertia for rotation about a point on the circumference is $\frac{3}{2}MR^2$.)
   (c) Derive the same result by applying $\dot{\mathbf{L}} = \mathbf{\Gamma}^{\text{ext}}$ to the rotation about the center of mass. (In this case you will find there is an extra unknown, the force of friction. You can eliminate this by applying Newton's second law to the motion of the center of mass. The moment of inertia for rotation about the center of mass is $\frac{1}{2}MR^2$.)

2. The following figure is a bird's-eye view of a smooth horizontal wire hoop that is forced to rotate at a fixed angular velocity ω about a vertical axis through the point A. A bead of mass m is threaded on the hoop and is free to move around it, with its position specified by the angle ϕ that it makes at the center with the diameter AB.
   (a) Find the Lagrangian for this system using ϕ as your generalized coordinate.
   (b) Using the Lagrangian equation of motion to show that the bead oscillates about the point B exactly like a simple pendulum.
   (c) What is the frequency of these oscillations if their amplitude is small?



3. Consider a pendulum suspended inside a railroad car that is being forced to accelerate with a constant acceleration a.
   (a) Write down the Lagrangian for the system and the equation of motion for the angle ϕ.
   (b) Find the equilibrium angle ϕ at which the pendulum can remain fixed (relative to the car) as the car accelerates.
   (c) Use the equation of motion to show that this equilibrium is stable. What is the frequency of small oscillation about this equilibrium position? Do you expect this result by considering the car as a non-inertia frame of reference? Explain.