1. **Delta function in the infinite square well**
Suppose you put a delta-function bump in the center of the infinite square well:
\[
H' = \alpha \delta(x - a/2),
\]
where \(\alpha\) is a constant.

   a) Find the first-order correction to the allowed energies. Explain why the energies are not perturbed for even \(n\).

   b) Find the first three nonzero terms in the expansion \((\psi_n^1)\) of the correction the ground state, \(\psi_1^1\).

Note that \(\psi_n^0(x) = \sqrt{\frac{2}{a}} \sin \left(\frac{n\pi}{a} x\right)\).

2. **The harmonic oscillator potential**
Find the first excited state of the harmonic oscillator by using the appropriate ladder operator
\[
a_\pm = \frac{1}{\sqrt{2\hbar m \omega}} (\mp i p + m \omega x)
\]
\[
\psi_0(x) = \left(\frac{m \omega}{\pi \hbar}\right)^{1/4} e^{-\frac{m \omega}{2\pi} x^2}
\]

3. **The energy-time uncertainty principle**
Consider the following equation
\[
\frac{d}{dt} \langle \hat{Q} \rangle = \frac{i}{\hbar} \langle [\hat{H}, \hat{Q}] \rangle + \langle \frac{\partial \hat{Q}}{\partial t} \rangle.
\]

   a) What does it tell you about the quantity “\(Q\)” if \(\hat{H}\) and \(\hat{Q}\) commute?

   b) Apply the given equation to the case of \(Q = 1\) … comment on your result.

   c) Apply the given equation to the case of \(Q = H\) … comment on your result.

   d) Apply the given equation to the case of \(Q = x\) … comment on your result.

   e) Apply the given equation to the case of \(Q = p\) … comment on your result.