Trigonometric Identities

$$\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi \qquad \cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$$

$$\cos\theta\cos\phi = \tfrac{1}{2}[\cos(\theta+\phi) + \cos(\theta-\phi)] \qquad \sin\theta\sin\phi = \tfrac{1}{2}[\cos(\theta-\phi) - \cos(\theta+\phi)]$$

$$\sin\theta\cos\phi = \tfrac{1}{2}[\sin(\theta+\phi) + \sin(\theta-\phi)]$$

$$\cos^2\theta = \tfrac{1}{2}[1+\cos 2\theta] \qquad \sin^2\theta = \tfrac{1}{2}[1-\cos 2\theta]$$

$$\cos\theta + \cos\phi = 2\cos\frac{\theta+\phi}{2}\cos\frac{\theta-\phi}{2} \qquad \cos\theta - \cos\phi = 2\sin\frac{\theta+\phi}{2}\sin\frac{\phi-\theta}{2}$$

$$\sin\theta \pm \sin\phi = 2\sin\frac{\theta\pm\phi}{2}\cos\frac{\theta\mp\phi}{2}$$

$$\cos^2\theta + \sin^2\theta = 1 \qquad \sec^2\theta - \tan^2\theta = 1$$

$$e^{i\theta} = \cos\theta + i\sin\theta \qquad \text{[Euler's relation]}$$

$$\cos\theta = \tfrac{1}{2}(e^{i\theta} + e^{-i\theta}) \qquad \sin\theta = \tfrac{1}{2i}(e^{i\theta} - e^{-i\theta})$$

Hyperbolic Functions

$$\cosh z = \tfrac{1}{2}(e^z + e^{-z}) = \cos(iz) \qquad \sinh z = \tfrac{1}{2}(e^z - e^{-z}) = -i\sin(iz)$$

$$\tanh z = \frac{\sinh z}{\cosh z} \qquad \text{sech} z = \frac{1}{\cosh z}$$

$$\cosh^2 z - \sinh^2 z = 1 \qquad \text{sech}^2 z + \tanh^2 z = 1$$

Series Expansions

$$f(z) = f(a) + f'(a)(z-a) + \tfrac{1}{2!}f''(a)(z-a)^2 + \tfrac{1}{3!}f'''(a)(z-a)^3 + \cdots \qquad \text{[Taylor's series]}$$

$$e^z = 1 + z + \tfrac{1}{2!}z^2 + \tfrac{1}{3!}z^3 + \cdots \qquad \ln(1+z) = z - \tfrac{1}{2}z^2 + \tfrac{1}{3}z^3 - \cdots [\,|z| < 1]$$

$$\cos z = 1 - \tfrac{1}{2!}z^2 + \tfrac{1}{4!}z^4 - \cdots \qquad \sin z = z - \tfrac{1}{3!}z^3 + \tfrac{1}{5!}z^5 - \cdots$$

$$\cosh z = 1 + \tfrac{1}{2!}z^2 + \tfrac{1}{4!}z^4 + \cdots \qquad \sinh z = z + \tfrac{1}{3!}z^3 + \tfrac{1}{5!}z^5 + \cdots$$

$$\tan z = z + \tfrac{1}{3}z^3 + \tfrac{2}{15}z^5 + \cdots [\,|z| < \pi/2] \qquad \tanh z = z - \tfrac{1}{3}z^3 + \tfrac{2}{15}z^5 - \cdots [\,|z| < \pi/2]$$

$$(1+z)^n = 1 + nz + \frac{n(n-1)}{2!}z^2 + \cdots [\,|z| < 1] \qquad \text{[binomial series]}$$

Some Derivatives

$$\frac{d}{dz}\tan z = \sec^2 z \qquad \frac{d}{dz}\tanh z = \sec h^2 z$$

$$\frac{d}{dz}\sinh z = \cosh z \qquad \frac{d}{dz}\cosh z = \sinh z$$

Some Integrals

$$\int \frac{dx}{c+ax^2} = \frac{1}{\sqrt{ac}}\arctan\left(x\sqrt{\frac{a}{c}}\right) \qquad \int \frac{dx}{c-ax^2} = \frac{1}{\sqrt{ac}}\arctan h\left(x\sqrt{\frac{a}{c}}\right)$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} \qquad \int \frac{dx}{\sqrt{a^2+x^2}} = \ln\left(x + \sqrt{a^2+x^2}\right)$$

$$\int \tan(ax)dx = -\frac{1}{a}\ln[\cos(ax)] \qquad \int \tanh(ax)dx = \frac{1}{a}\ln[\cosh(ax)]$$

$$\int \frac{dx}{ax^2+bx} = \frac{\ln x}{b} - \frac{\ln(b+ax)}{b} \qquad \int \frac{xdx}{ax^2+b} = \frac{\ln(b+ax)}{a}$$

$$\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|x + \sqrt{x^2-a^2}\right| \qquad \int \frac{xdx}{\sqrt{a^2+x^2}} = \sqrt{a^2+x^2}$$

$$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\arccos\frac{a}{x} \qquad \int \ln x dx = x\ln(x) - x$$

$$\int \frac{dx}{(1+x^2)^{3/2}} = \frac{x}{(1+x^2)^{1/2}}$$

$$\int \sqrt{\frac{a+x}{b-x}} = -\sqrt{(a+x)(b-x)} - (a+b)\arcsin\sqrt{\frac{b-x}{a+b}}$$

$$\int_0^1 \frac{dx}{\sqrt{1-x^2}\sqrt{1-mx^2}} = K(m), \text{ complete elliptical integral of } 1^{\text{st}} \text{ kind}$$

Vector Calculus

$$\nabla f = \hat{\mathbf{x}}\frac{\partial f}{\partial x} + \hat{\mathbf{y}}\frac{\partial f}{\partial y} + \hat{\mathbf{z}}\frac{\partial f}{\partial z} \quad \text{[Cartesian]}$$

$$= \hat{\mathbf{r}}\frac{\partial f}{\partial r} + \hat{\boldsymbol{\theta}}\frac{1}{r}\frac{\partial f}{\partial \theta} + \hat{\boldsymbol{\phi}}\frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi} \quad \text{[spherical polars]}$$

$$= \hat{\boldsymbol{\rho}}\frac{\partial f}{\partial \rho} + \hat{\boldsymbol{\phi}}\frac{1}{\rho}\frac{\partial f}{\partial \phi} + \hat{\mathbf{z}}\frac{\partial f}{\partial z} \quad \text{[cylindrical polars]}$$

$$\nabla \times \mathbf{A} = \hat{\mathbf{x}}\left(\frac{\partial}{\partial y}A_z - \frac{\partial}{\partial z}A_y\right) + \hat{\mathbf{y}}\left(\frac{\partial}{\partial z}A_x - \frac{\partial}{\partial x}A_z\right)$$

$$+ \hat{\mathbf{z}}\left(\frac{\partial}{\partial x}A_y - \frac{\partial}{\partial y}A_x\right) \quad \text{[Cartesian]}$$

$$= \hat{\mathbf{r}}\frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(\sin\theta A_\phi) - \frac{\partial}{\partial \phi}A_\theta\right] + \hat{\boldsymbol{\theta}}\left[\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}A_r - \frac{1}{r}\frac{\partial}{\partial r}(rA_\phi)\right]$$

$$+ \hat{\boldsymbol{\phi}}\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial}{\partial \theta}A_r\right] \quad \text{[spherical polar]}$$

$$= \hat{\boldsymbol{\rho}}\left[\frac{1}{\rho}\frac{\partial}{\partial \phi}A_z - \frac{\partial}{\partial z}A_\phi\right] + \hat{\boldsymbol{\phi}}\left[\frac{\partial}{\partial z}A_\rho - \frac{\partial}{\partial \rho}A_z\right]$$

$$+ \hat{\mathbf{z}}\frac{1}{\rho}\left[\frac{\partial}{\partial \rho}(\rho A_\phi) - \frac{\partial}{\partial \phi}A_\rho\right] \quad \text{[cylindrical polar]}$$

$$\nabla \cdot \mathbf{A} = \frac{\partial}{\partial x}A_x + \frac{\partial}{\partial y}A_y + \frac{\partial}{\partial z}A_z \quad \text{[Cartesian]}$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}A_\phi \quad \text{[spherical polars]}$$

$$= \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial}{\partial \phi}A_\phi + \frac{\partial}{\partial z}A_z \quad \text{[cylindrical polars]}$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} \quad \text{[Cartesian]}$$

$$= \frac{1}{r}\frac{\partial^2}{\partial r^2}(rf) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \phi^2} \quad \text{[spherical polars]}$$

$$= \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2} \quad \text{[cylindrical polars]}$$

Newton's 2nd Law in Various Coordinate Systems

Vector Form	Cartesian (x,y,z)	2D Polar (r,ϕ)	Cylindrical Polar (ρ,ϕ,z)
$\vec{F} = m\ddot{\vec{r}}$	$\begin{cases} F_x = m\ddot{x} \\ F_y = m\ddot{y} \\ F_z = m\ddot{z} \end{cases}$	$\begin{cases} F_r = m(\ddot{r} - r\dot{\phi}^2) \\ F_\phi = m(r\ddot{\phi} + 2\dot{r}\dot{\phi}) \end{cases}$	$\begin{cases} F_\rho = m(\ddot{\rho} - \rho\dot{\phi}^2) \\ F_\phi = m(\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi}) \\ F_z = m\ddot{z} \end{cases}$

Derivative elements	Coordinate System
$d\vec{l} = dx\ \hat{i} + dy\ \hat{j} + dz\ \hat{k}$ $dV = dx\ dy\ dz$	Cartesian
$d\vec{l} = dr\ \hat{r} + r\ d\phi\ \hat{\phi} + dz\ \hat{z}$ $dV = r\ dr\ d\phi\ dz$	Cylindrical
$d\vec{l} = dr\ \hat{r} + rd\theta\ \hat{\theta} + r\sin\theta d\phi\ \hat{\phi}$ $dV = r^2\sin\theta\ dr\ d\theta\ d\phi$	Spherical