Physics Preliminary Exam Spring 2012
Paper 2 – Quantum Mechanics
Feb 1st 2012
3-5pm

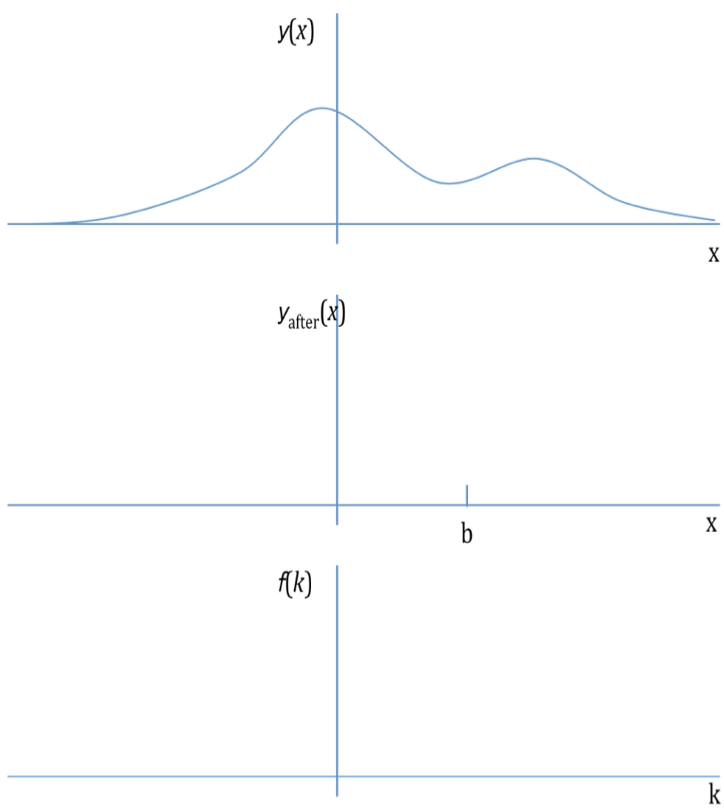
Attempt 2 out of the 3 problems

Question 1 (20 pts)

The position of a particle is determined to be $x=b$. **Prior to making the measurement** the particle was in the state $\psi(x)$ depicted below (top graph).

a) Sketch the new state for the particle, $\psi_{after}(x)$, immediately after the measurement on the middle graph.

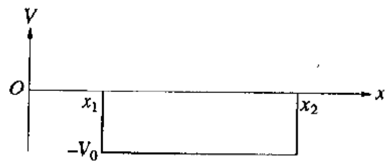
b) Sketch $\phi_{after}(k)$ for the particle after the measurement on the lower graph.



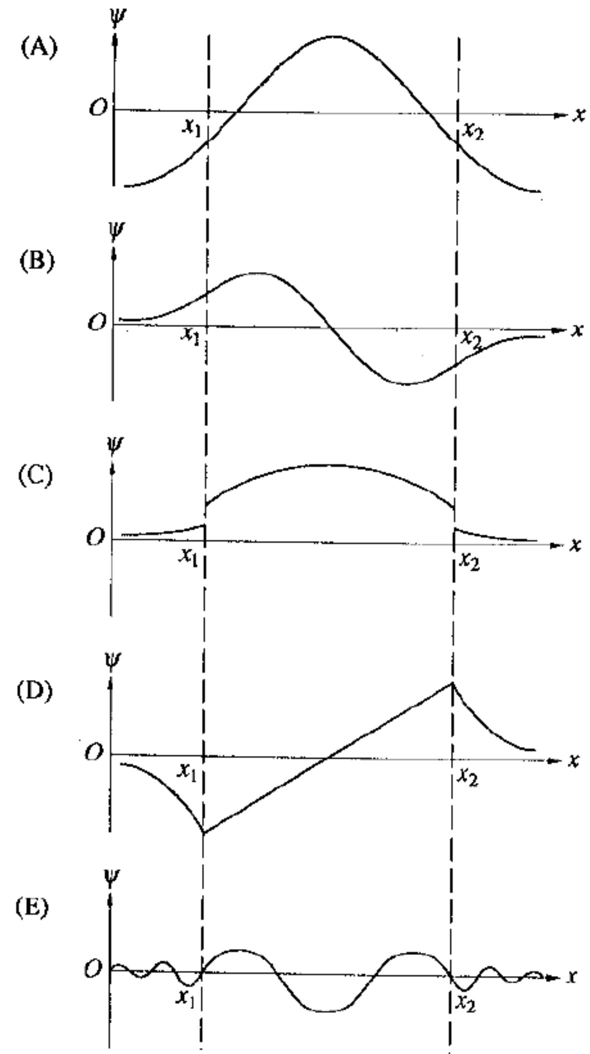
(c) Consider the following GRE question from an old exam.

What is the best answer?

Why are the other ones incorrect?



29. An attractive, one-dimensional square well has depth V_0 as shown above. Which of the following best shows a possible wave function for a bound state?



Question 2 (20pts)

Sequential measurements:

An operator $\hat{A}$, representing observable A, has two normalized eigenstates ψ_1 and ψ_2, with eigenvalues a_1 and a_2, respectively.

Operator $\hat{B}$, representing observable B, has two normalized eigenstates ϕ_1 and ϕ_2, with eigenvalues b_1 and b_2.

The eigenstates are related by:

$$\psi_1 = \frac{(3\phi_1 + 4\phi_2)}{5} \qquad\qquad \psi_2 = \frac{(4\phi_1 - 3\phi_2)}{5}$$

a) Observable A is measured, and the value a_1 is obtained. What is the state of the system immediately after this measurement?

b) If B is now measured, what are the possible results, and what are their probabilities?

c) Right after the measurement of B, A is measured again. What is the probability of getting a_1?

Question 3 (20 pts)

One can study the harmonic oscillator using raising and lowering operators.

a) Use the lowering operator,

$$\hat{a}_{-} = \frac{1}{\sqrt{2m\omega\hbar}}\left(+i\hat{p} + m\omega\hat{x}\right),$$

on the ground state wavefunction, $\psi_0(x)$, to show that:

$$\psi_0(x) = Ae^{-\frac{m\omega}{2\hbar}x^2}$$

b) Normalize $\psi_0(x) = Ae^{-\frac{m\omega}{2\hbar}x^2}$.

c) Compute $\langle x \rangle$ and $\langle p \rangle$ for $\psi_0(x)$.