Physics Preliminary Exam Fall 2010
Paper 2 – Quantum Mechanics
Sept 15th 2010
6-8pm

Attempt 2 out of the 3 Questions

Question 1. Consider one-dimensional particle with the potential energy

$$V(x) = \{ \begin{array}{ll} \infty, & x < 0, \quad x > a \\ 0, & 0 < x < a/2 \\ V_0, & a/2 < x < a \end{array}$$

which describes an infinite square well outside the range $0 < x < a$. But inside the range $0 < x < a$, the potential is a step function. For an eigenenergy $E < V_0$, the eigenstate has the following form:

$$\psi_n(x) = \{ \begin{array}{ll} C\sin(kx), & 0 < x < a/2 \\ Ae^{\kappa x} + Be^{-\kappa x}, & a/2 < x < a \end{array}$$

(a) express k and κ in terms of the energy E and potential V_0. (b) using the boundary condition $\psi_n(a) = 0$ to express B in terms of A. (c) write down the two boundary conditions at $x = a/2$.

Question 2. Quantum harmonic oscillator can be described by the Hamiltonian $\hat{H} = \hbar\omega_0(\hat{a}_+\hat{a}_- + 1/2)$. Here, $\hat{a}_\pm$ are the raising and lowering operators defined as

$$\hat{a}_\pm = \frac{1}{\sqrt{2\hbar m\omega_0}}(\mp i\hat{p} + m\omega_0\hat{x})$$

respectively in terms of the coordinate operator $\hat{x}$ and the momentum operator $\hat{p}$. (a) derive the communtation relation $[\hat{a}_-,\, \hat{a}_+]$ using the commutation relation $[\hat{x}, \hat{p}] = i\hbar$. (b) using the result in (a), derive $[\hat{H}, \hat{a}_-]$. (c) Given that the Hermition conjugate of $\hat{p}$ is $\hat{p}$ and the Hermition conjugate of $\hat{x}$ is $\hat{x}$, show that the Hermition conjugate of $\hat{a}_+$ is $\hat{a}_-$.

Question 3

Electrons impinge on a double slit and form an interference pattern on a far-away screen with spatial period s. The slits have equal width that is much smaller than the electrons deBroglie wavelength. The contrast C of the interference pattern is defined as , $C = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$

where I_{max} and I_{min} are the maximum and minimum intensity on the screen.

a) Assume that we have a way of changing the phase of the wavefunction at each of the slits without changing its amplitude. If we change the phase of the wavefunction at slit 1 by ϕ_1, and the phase of the wave function at slit 2 by ϕ_2, what happens to the electron interference pattern on the screen? (Position and contrast.)

b) Now assume that we do not change the phases of the wavefunctions at the slits, but instead make slit 1 half as wide as slit 2. What happens to the interference pattern on the screen? (Position and contrast.)

c) What happens to the interference pattern if we replace the electrons impinging on the double-slit by muons of the same energy? If the interference pattern changes, specify the change quantitatively. The mass of the muon is 207 times larger than that of the electron, while the charge is the same. Both the electrons and the muons are assumed to be non-relativistic.