Preliminary Exam

Electricity and Magnetism

Fall 2021
UC Merced

Complete two of the following three questions.

Problem 1 (20 points).
a) A circle of radius a lies in the xy-plane and is centered at the origin. It carries a uniform linear charge density $\lambda > 0$. Sketch the electric field lines as they appear in the yz-plane.

b) An electric dipole $\mathbf{p} = p\hat{\mathbf{z}}$ points in the z-direction and lies at $(x, y, z) = (0, 2a, 0)$. What direction will the dipole rotate: clockwise, counterclockwise, or not at all?

c) Now imagine that the circle in part a has zero charge density, but instead possesses a current I, circulating clockwise in the xy-plane. In a separate figure, sketch the magnetic field lines in the yz-plane.

d) A magnetic dipole $\mathbf{m} = m\hat{\mathbf{z}}$ points in the z-direction and lies at $(x, y, z) = (0, 2a, 0)$. What direction will the dipole rotate, clockwise, counterclockwise, or not at all?

e) What is the dipole moment $\mathbf{M}$ (magnitude and direction) of the current loop itself?

Problem 2 (20 points). A series of short questions (5 points each).

a) Find a potential $V(x, y, z)$ for the electric field $\mathbf{E} = ay\hat{\mathbf{x}} + ax\hat{\mathbf{y}} + b\hat{\mathbf{z}}$, where a and b are constants.

b) Find the volume current $\mathbf{J}$ such that the following electric and magnetic fields satisfy Maxwell's equations

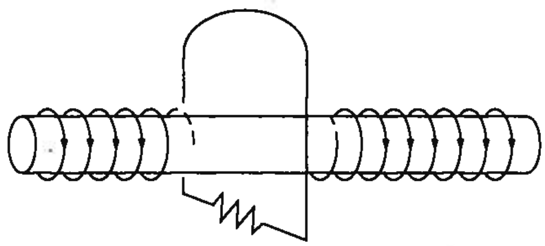
$$\mathbf{E} = \frac{It}{4\pi\epsilon_0 r^2}\hat{\boldsymbol{r}}, \qquad \mathbf{B} = 0, \tag{1}$$

where I is constant.

c) A point charge q is placed a distance $2d$ above the surface of an infinite grounded conducting slab of thickness d. (i) What is the magnitude of the force on q? Is it attracted to or repelled from the conductor? (ii) What is the electric field *below* the conducting slab? (Don't forget about boundary conditions!)

d) Two parallel infinite wires carry the same current I propagating in the same direction. Will the force between them be attractive or repulsive? Justify your answer in terms of the Lorentz force law. (You needn't compute the magnitude of the force, just justify the direction of the force.)

Problem 3 (20 points).
A long solenoid of radius a, carrying n turns per unit length, is looped by a wire with resistance R, as shown below.



a) Assume the currents in the solenoid and loop are initially zero. If the current in the solenoid is now increased at a constant rate ($dI/dt = k$), what current flows in the loop, and which way (left or right) does it pass through the resistor?

b) Assume now that the solenoid initially has a current I and the loop initially has zero current. Next, the solenoid is pulled out of the loop, turned around, and reinserted. What total charge passes through the resistor? (You don't need to worry about the *sign* of the charge.)

VECTOR DERIVATIVES

Cartesian. $d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$; $\quad d\tau = dx\,dy\,dz$

Gradient : $$\nabla t = \frac{\partial t}{\partial x}\hat{\mathbf{x}} + \frac{\partial t}{\partial y}\hat{\mathbf{y}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

Curl : $$\nabla \times \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{\mathbf{z}}$$

Laplacian : $$\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

Spherical. $d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}$; $\quad d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi$

Gradient : $$\nabla t = \frac{\partial t}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

Curl : $$\nabla \times \mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial \phi}\right]\hat{\mathbf{r}}$$
$$+ \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial \theta}\right]\hat{\boldsymbol{\phi}}$$

Laplacian : $$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial t}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial \phi^2}$$

Cylindrical. $d\mathbf{l} = ds\,\hat{\mathbf{s}} + s\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}$; $\quad d\tau = s\,ds\,d\phi\,dz$

Gradient : $$\nabla t = \frac{\partial t}{\partial s}\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

Curl : $$\nabla \times \mathbf{v} = \left[\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{\mathbf{s}} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial \phi}\right]\hat{\mathbf{z}}$$

Laplacian : $$\nabla^2 t = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}$$