Classical Mechanics (Fall 2015)

Please choose 2 (and only 2) problems to answer. On your answer sheet, please clearly state which two problems you choose.

Problem 1: Two particle system

Two particles with position vectors, $\overrightarrow{r}_1$ and $\overrightarrow{r}_2$, and masses, m_1 and m_2, attract each other with the force $\overrightarrow{F}_{12} = -\overrightarrow{F}_{21} = k(\overrightarrow{r}_1 - \overrightarrow{r}_2)$, where $k > 0$ and constant.

a) Reduce this two particle problem to the equivalent one body problem.

b) Show that the relative motion occurs in a plane.

c) Solve the equation of motions and show that in the center of mass system the paths of the two particles are ellipses with centers in the origin.

d) How are the periods of circulation related to the path parameters and the masses of the particles?

Problem 2: Bead on a spinning wire
Consider a bead of mass m sliding without friction on a wire that is bent in the shape of a parabola and is being spun with a constant angular velocity ω about its vertical axis, as shown in Figure 1. Use cylindrical polar coordinates and let the equation of the parabola be $z = k\rho^2$.

1) Write down the Lagrangian in terms of ρ as the generalized coordinate.

2) Find the equation of motion of the bead and determine whether there are positions of equilibrium, that is, values of ρ at which the bead can remain fixed, without sliding up or down the spinning wire.

3) Discuss the stability of any equilibrium positions you find.

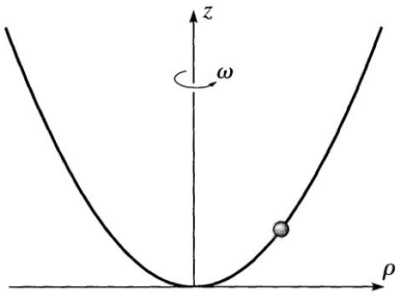

Figure 1: Bead on parabolically shaped wire.

Problem 3: Motion of an oil tank

A tall oil tank (considered as a hollow cylinder with infinitely thin walls) of mass M and radius r is rolling down an inclined plane at angle θ without slip.

a) Write down the Lagrangian of the system in terms of the height h, rotation rate ω, and moment of inertia I.

b) Find the moment of inertia I in terms of M and r. Write down the equation of motion for $\dot{\varphi} = \omega$.

c) Find the acceleration of the tank in term of θ.

d) Considering the tank half-filled with water (of mass m) and assuming that the water surface remains flat (NOT dragged along by the rotating tank) during the motion, find the acceleration of the tank.

e) Indeed, the water has an inclined surface in an accelerating frame of reference due to its inertia. Noting that the pressure p of the water near its surface depends on the height l and density ρ as $p = \rho g l$, find the angle of this inclined surface in terms of the acceleration rate. How does this inclined water surface affect your previous result on the acceleration of the tank?