Prelim Exam

Quantum Mechanics

Fall 2014

Answer any 2 out of the three questions below. (2 hours)

Question 1: Raising and lowering operators in the harmonic oscillator potential

Given $\psi_0(x) = Ae^{-\frac{m\omega}{2h}x^2}$, normalize to find A.

Use the appropriate raising or lowering operator to obtain $\psi_1(x)$ (you don't need to normalize your answer…if you do it correctly, it already is).

$$a_\pm = \frac{1}{\sqrt{2\hbar m\omega}}\left(\mp ip + m\omega x\right)$$

Definite integrals

$$\int_0^\infty e^{-ax^2}dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty x^n e^{-x/a}dx = n!a^{n+1}$$

$$\int_0^\infty x^{2n} e^{-x^2/a^2}dx = \sqrt{\pi}\frac{(2n)!}{n!}\left(\frac{a}{2}\right)^{2n+1}$$

$$\int_0^\infty x^{2n+1} e^{-x^2/a^2}dx = \frac{n!}{2}a^{2n+2}$$

Conceptual questions

For all parts of this problem, consider a system where we can measure two quantum mechanical observables, "color" (represented by the $\hat{C}$ operator) and "size" (represented by the $\hat{S}$ operator). The "color" operator has three eigenvalues (red, green, and blue) and three corresponding eigenstates ($|r\rangle$,$|g\rangle$, and $|b\rangle$). The "size" operator also has three eigenvalues (small, medium, and large) with three corresponding eigenstates ($|s\rangle$,$|m\rangle$, and $|l\rangle$).

We wish to understand how color and size interact. Since we cannot see the "color" or "size" of our particles with our eyes, we build a lab with a ColorTron™ device which measures the "color" eigenvalue of a particle and a SizeUp™ device which measures the particle's "size" eigenvalue.

Part I: In the setup just described, an experimenter measures the color of particles and then immediately runs all particles which measured red through the ColorTron. **What are the possible results of this second measurement?**

A. The only possible measurement is red with a 100% probability of measurement.
B. Red, green, and blue can be measured with equal probabilities.
C. Red, green, and blue can be measured, but their associated probabilities of measurement cannot be determined from the information given.

D. There is not enough information to answer this questions.

Part II: 1000 red particles are immediately run through the SizeUp which measures the small eigenvalue 200 times, the medium eigenvalue 300 times, and the large eigenvalue 500 times. **Which one of the following could be a valid representation of the $|r\rangle$ state in the "size" basis?**

A. $200|s\rangle + 300|m\rangle + 500|l\rangle$

B. $\frac{1}{10}\left(2|s\rangle + 3|m\rangle + 5|l\rangle\right)$

C. $\frac{1}{\sqrt{10}}\left(e^{i\alpha_1}\sqrt{2}|s\rangle + e^{i\alpha_2}\sqrt{3}|m\rangle + e^{i\alpha 3}\sqrt{5}|l\rangle\right)$

D. $\frac{1}{10}e^{i\alpha}\left(\sqrt{2}|s\rangle + \sqrt{3}|m\rangle + \sqrt{5}|l\rangle\right)$

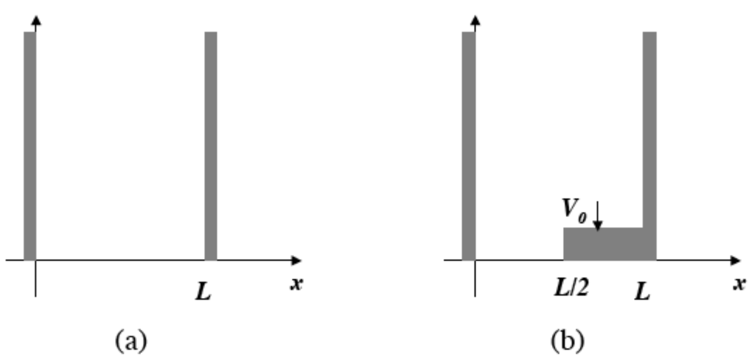
E. None of the above.

Part III: 1000 medium particles are immediately run through the ColorTron and then immediately run through the SizeUp without looking at which colors were produced. If "color" and "size" commute, **what can you say about the results from the SizeUp measurements?**

A. All three "size" eigenvalues could be measured.
B. Most but not all of the particles will be measured as medium.
C. All particles will be measured as medium.
D. The results are probabilistic and cannot be predicted.
E. The "color" measurement will affect the "size" measurement.

Question 2: Square well

Consider a particle in a one dimensional infinite square well. The length of the well is L.



1. Calculate the first three eigenstates $\psi_1(x), \psi_2(x), \psi_3(x)$ and their eigenvalues of the infinite square well.

2. With an initial wave function $\psi(x,0)=\psi_1(x)+\frac{1}{2}\psi_2(x)$, write down the time evolution $\psi(x,t)$

3. In $\psi(x,t)$, what's the probability of the particle in state $\psi_1(x),\psi_2(x),\psi_3(x)$, respectively?

Question 3: Operators and eigenvectors

Given the two operator matrices:

$$H=\hbar\omega\begin{pmatrix}2&0&0\\0&3&0\\0&0&1\end{pmatrix};\qquad G=\gamma\begin{pmatrix}0&2&0\\2&0&0\\0&0&1\end{pmatrix}$$

a) Calculate $\left[G,H\right]$

b) Do G and H represent compatible observables (explain)?

The eigenvalues of G are g=-2,g=1, and g=2.

Two of the eigenvectors are given by

$$\text{for } g=1;\quad \begin{pmatrix}0\\0\\1\end{pmatrix}, \text{ and for } g=2;\quad \begin{pmatrix}1\\1\\0\end{pmatrix}$$

c) Find the remaining eigenvector.

If the state at time t=0 is given by:

$$\left|\psi(0)\right\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}-1\\1\\0\end{pmatrix}$$

d) What is the $\left|\psi(t)\right\rangle$ at some later time?