Jan. 2020 CM Prelim

UC Merced

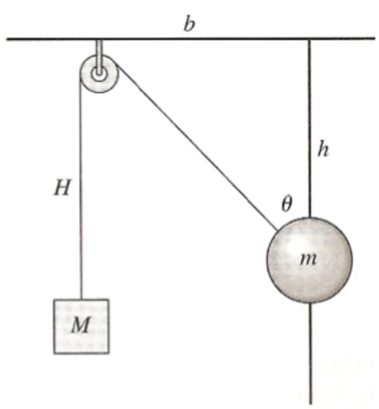
Attempt only 2 out of the 3 problems

1. A metal ball (mass m) with a hole through it is threaded on a frictionless vertical rod. A massless string (length l) attached to the ball runs over a massless, frictionless pulley and supports a block of mass M, as shown in the figure.

   (a) Write down the potential energy in terms of the angle θ.

   (b) Does this system have an equilibrium position?

   (c) Discuss the stability of any equilibrium positions.



2. Consider the modified Atwood machine in the figure. The two weights on the left have equal masses m and are connected by a massless spring of force constant k. The weight on the right has mass $M = 2m$, and the pulley is massless and frictionless. The coordinate x is the extension of the spring from its equilibrium length; that is, the length of the spring is $l_e + x$ where l_e is the equilibrium length. Solve them for the following initial conditions: You hold the mass M fixed with the whole system in equilibrium and . Describe the motion and find the frequency with which x oscillates.

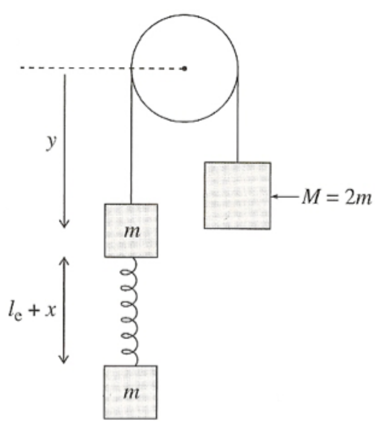
   (a) Write down the Hamiltonian equations of motion.

   Solve them for the following initial conditions:

   (b) You hold the mass M fixed with the whole system in equilibrium and

   (c) still holding M fixed, you pull the lower mass m down a distance x_0, and at $t = 0$ you let go of both masses.

   (d) Describe the motion and find the frequency with which x oscillates.



3. A certain rocket carries a fraction α of its initial mass as fuel. (That is, the mass of the fuel is αm_o.)

   (a) What is the rocket's final speed, accelerating from rest in free space, if it burns all its fuel in a single stage? Express your answer as a multiple of exhausting speed v_{ex}.

   (b) Suppose instead it burns the fuel in two stages as follows: In the first stage it burns a mass of $\frac{\alpha}{2}m_o$ of fuel. It then jettisons the first-stage fuel tank, which has a mass of βm_o, and then burns the remaining $\frac{\alpha}{2}m_o$ of fuel. Find the final speed in this case, assuming the same value of v_{ex} throughout.

   (c) Prove that the rocket will always achieve a higher speed in the latter (two-stage) case if $\beta > 0$.