Jan. 2020 EM Prelim

UC Merced

Attempt only 2 out of the 3 problems

1. Consider a charged spherical shell of radius R with constant surface charge density σ.

   (a) Find $\vec{\mathbf{E}}$ and V everywhere. Make sure your solution satisfies $V(r \to \infty) = 0$.

   (b) Compute the total amount of work needed to assemble the shell of charge, using whichever method you like.

   (c) How much work would it take to bring an additional charge, q, from 'far away' ($r = \infty$) to the center of the sphere?

   (d) Suppose we filled the *inside* of the sphere with a linear dielectric with permittivity ϵ_1. Would this change your answers to (a-c) above? Explain. (You don't need to solve the problem, just indicate what would happen.)

2. Suppose we have a circular, flexible loop of wire in the x-y plane. This loop has radius $S(t) = vt$, which is changing in time because we are stretching it out. The loop is placed in a constant external magnetic field:

$$\vec{\mathbf{B}} = B_x\hat{\mathbf{x}} + B_y\hat{\mathbf{y}} + B_z\hat{\mathbf{z}} \tag{1}$$

(a) What is the flux of the external magnetic field through the loop of wire as a function of $S(t)$?

(b) If the loop has a resistance R, at what rate does it dissipate energy?

(c) The induced current also creates an extra magnetic field. Which direction does this extra magnetic field point at the center of the loop? You may assume $v > 0$. (I don't care about magnitude, just the direction and sign!)

3. Maxwell's equations before Maxwell were given by:

$$\vec{\nabla} \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0} \tag{2}$$

$$\vec{\nabla} \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t} \tag{3}$$

$$\vec{\nabla} \cdot \vec{\mathbf{B}} = 0 \tag{4}$$

$$\vec{\nabla} \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{J}} \tag{5}$$

(a) Prove, using vector calculus identities, that these equations *must* be incomplete for time carrying charges and/or currents.

(b) Show that these pre-Maxwell solutions can *not* support a (complex) plane wave solution of in free space ($\rho = 0$, $\vec{\mathbf{J}} = 0$). For simplicity you may assume the following form for the travelling wave:

$$\vec{\mathbf{E}} = E_0 \ \hat{\mathbf{x}} \ e^{ik(z-vt)} \tag{6}$$

$$\vec{\mathbf{B}} = B_0 \ \hat{\mathbf{y}} \ e^{ik(z-vt)} \tag{7}$$