Jan. 2020 QM Prelim

UC Merced

Attempt only 2 out of the 3 problems

1. Shown below is the normalized time-independent wavefunction for hydrogen.

$$\psi_{nlm}(r,\theta,\phi) = \sqrt{\left(\frac{2}{na}\right)^3 \frac{(n-l-1)!}{2n[(n+l)!]^3}} e^{-\frac{r}{na}} \left(\frac{2r}{na}\right)^l \left[L_{n-l-1}^{2l+1}\left(\frac{2r}{na}\right)\right] Y_l^m(\theta,\phi)$$

   (a) Copy the part(s) of the equation representing the "radial" component of the wavefunction.

   (b) Copy the part(s) of the equation representing the "angular" components of the wavefunction.

   (c) What is the origin of $\sqrt{\left(\frac{2}{na}\right)^3 \frac{(n-l-1)!}{2n[(n+l)!]^3}}$?

   (d) Describe each of the following variables using a sentence or two:

      i. r,

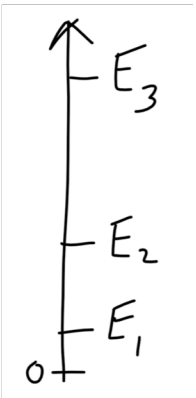
      ii. n,

      iii. a,

      iv. l,

      v. m.

2. The figure to the right illustrates the lowest 3 energy levels of the bound-state system having an infinite number of energy levels.



The levels are labeled in the standard way where the nth eigenstate $|n\rangle$ has an energy eigenvalue of E_n.

(a) If possible, give an example of a quantum state where the position probability distribution is time-independent.

(b) Explain why the quantum state is time-independent, or explain why it is not possible to give such an example.

(c) If it is possible to write a time-dependent state, what could you change in order to make it vary more quickly with respect to time.

(d) Explain your reasoning for part c.

3. Suppose you have a quantum dot whose dimensions are approximately $a = 10$ nm on a side. For simplicity, we will assume that the behavior of a particle having a mass of about $m = 0.1$ m_e confined within a quantum dot can be modeled by an infinite 1-d potential well.

   (a) Sketch the potential energy as a function of position.

   (b) What are the solutions for the wave function, $\psi_n(x)$ - (please express as a formula ... do not plug in the values for a and m)?

   (c) What are the energy eigenvalues E_n for this case - (please express as a formula ? do not plug in the values for a and m)?

   (d) NOW, please insert the values for a and m. What is the energy of the ground state? ($m_e = 9.1 \times 10^{-31}$ kg; $\hbar = 1.05 \times 10^{-34}$ J·s)