Name:________________________________

A finite well

Consider the finite well defined by

$$V(x) = \begin{cases} -V_{0}, & -a \le x \le a, \\ 0, & |x| > a, \end{cases}$$

a) Sketch the potential energy as a function of position.

b) What are the **general** solutions for the wave function in each of the three regions? The general solution means that you need not normalize, apply boundary conditions or find the eigenvalues.

Region 1: $x < -a$,

Region 2: $-a \le x \le a$,

Region 3: $x > a$

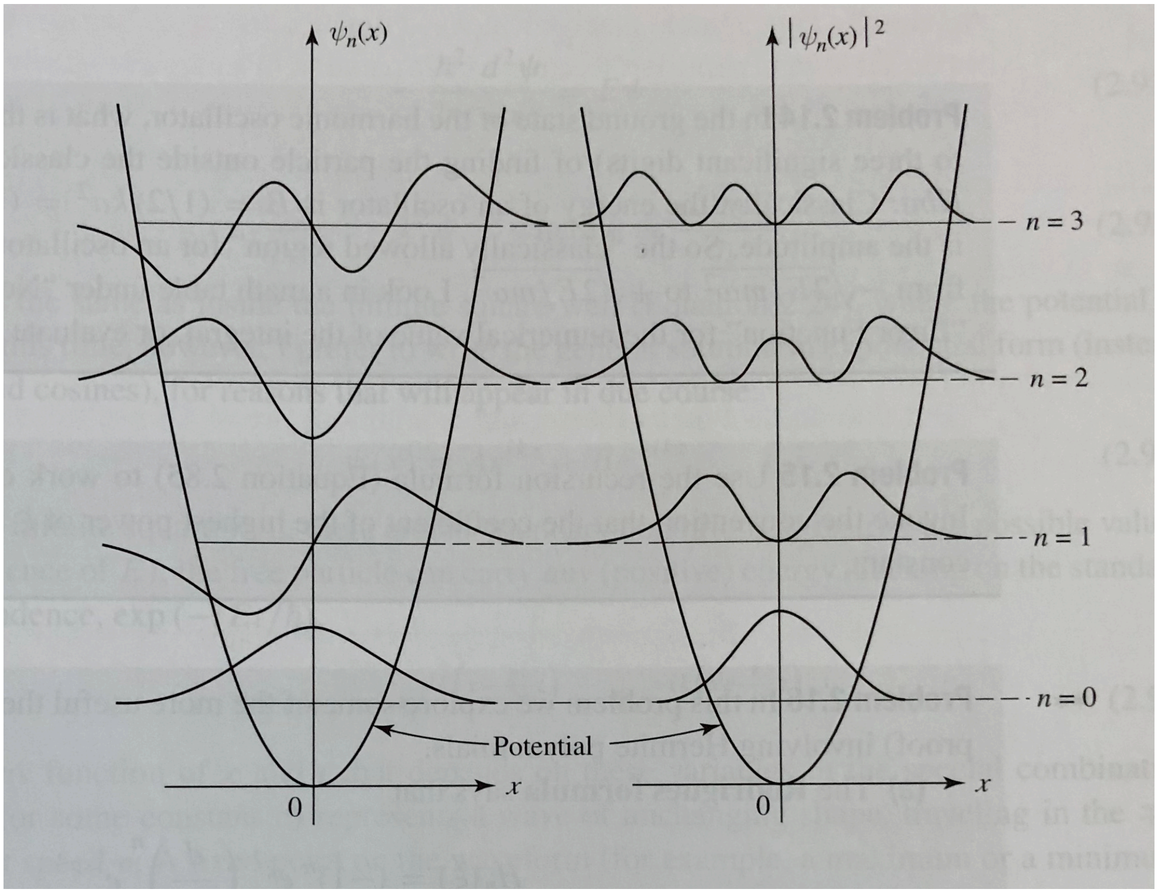
c) Describe the behavior of the system of the limiting case of a wide, deep well.

d) Describe the behavior of the system of the limiting case of a narrow, shallow well.

Name:________________________________

States of the harmonic oscillator

The figure to the left illustrates 4 eigenstates of the harmonic oscillator.



The levels are labeled in the standard way where the nth eigenstate $|n\rangle$ has an energy eigenvalue of $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$.

a) Suppose one prepares this particular system in its ground state ψ_0; $n = 0$. If one then measures the energy of the system at some later time what is the most probable result of that measurement?

b) Suppose one prepares this particular system in a 50/50 superposition of the $n = 0$ and $n = 2$ states.

   a. Is this a stationary state? Please explain.

   b. What is the probability of measuring each of the following energies (explain your reasoning as well)

      i. $E = \frac{\hbar\omega}{2}$?
      ii. $E = \frac{3\hbar\omega}{2}$?
      iii. $E = \frac{5\hbar\omega}{2}$?
      iv. $E = \frac{7\hbar\omega}{2}$?

c) Give an example of a combination of harmonic oscillator states which, when allowed to evolve in time, will flop between the states with a frequency of $\Omega = \hbar\omega$. Please explain your choice.

d) Give an example of a state where the probability of finding the particle in the vicinity of $x = 0$ is negligible. Please explain your choice.

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Heisenberg uncertainty principle

The Heisenberg uncertainty principle is frequently expressed as follows:

$$\sigma_x \sigma_p \geq \frac{\hbar}{2}$$

a) In this expression, is it correct to refer to position and momentum as *compatible* observables? Please explain why or why not.

b) Explain, by describing the behavior of wavefunctions, why one cannot simply measure the position of the object using a really good ruler, then measure the momentum using a really good *momentometer* so that the Heisenberg uncertainty principle is violated.

c) Is it possible, in principle, to conduct an experiment where $\sigma_x < \frac{\hbar}{10}$? Please explain why or why not.

d) Please give an example of two quantities, other than position and momentum, which do not commute