A finite well
Consider the finite well defined by
\[ V(x) = \begin{cases} -V_0, & -a \leq x \leq a, \\ 0, & |x| > a, \end{cases} \]

a) Sketch the potential energy as a function of position.

b) What are the general solutions for the wave function in each of the three regions? The general solution means that you need not normalize, apply boundary conditions or find the eigenvalues.

Region 1: \( x < -a, \)
Region 2: \( -a \leq x \leq a, \)
Region 3: \( x > a \)

c) Describe the behavior of the system of the limiting case of a wide, deep well.

d) Describe the behavior of the system of the limiting case of a narrow, shallow well.
States of the harmonic oscillator

The figure to the left illustrates 4 eigenstates of the harmonic oscillator. The levels are labeled in the standard way where the nth eigenstate $|n\rangle$ has an energy eigenvalue $E_n = (n + \frac{1}{2}) \hbar \omega$.

a) Suppose one prepares this particular system in its ground state $\psi_0$; $n = 0$. If one then measures the energy of the system at some later time what is the most probable result of that measurement?
b) Suppose one prepares this particular system in a 50/50 superposition of the \( n = 0 \) and \( n = 2 \) states.

a. Is this a stationary state? Please explain.

b. What is the probability of measuring each of the following energies (explain your reasoning as well)
   i. \( E = \frac{\hbar \omega}{2} \)
   ii. \( E = \frac{3\hbar \omega}{2} \)
   iii. \( E = \frac{5\hbar \omega}{2} \)
   iv. \( E = \frac{7\hbar \omega}{2} \)

c) Give an example of a combination of harmonic oscillator states which, when allowed to evolve in time, will flop between the states with a frequency of \( \Omega = \hbar \omega \). Please explain your choice.

d) Give an example of a state where the probability of finding the particle in the vicinity of \( x = 0 \) is negligible. Please explain your choice.
Heisenberg uncertainty principle
The Heisenberg uncertainty principle is frequently expressed as follows:

\[ \sigma_x \sigma_p \geq \frac{\hbar}{2} \]

a) In this expression, is it correct to refer to position and momentum as *compatible* observables? Please explain why or why not.

b) Explain, by describing the behavior of wavefunctions, why one cannot simply measure the position of the object using a really good ruler, then measure the momentum using a really good *momentometer* so that the Heisenberg uncertainty principle is violated.

c) Is it possible, in principle, to conduct an experiment where \( \sigma_x < \frac{\hbar}{10} \)? Please explain why or why not.

d) Please give an example of two quantities, other than position and momentum, which do not commute