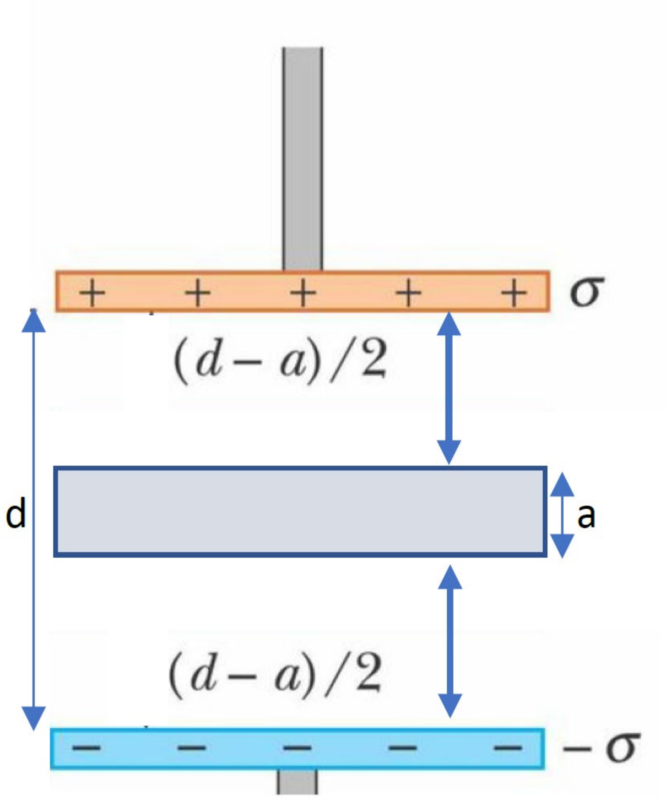
Preliminary exam

Electricity and Magnetism

Please choose 2 out of the 3 problems on the following pages. Please clearly state which problems you are choosing. Do NOT attempt all of the problems. If more than 2 problems are attempted, only the first two will be considered for a grade.

1. The diagram below illustrates a parallel plate capacitor with plate separation d and plate area A. An uncharged metallic slab of thickness a is inserted midway between the plates.



a. Draw any charges that result on the surfaces of the metal slab. You need only consider the surfaces of the slab that directly face the plates of the capacitor. (3 pts)

b. The metal slab only serves to make an electrical connection between the two charged surfaces of the slab. Therefore, redraw the diagram above as an equivalent circuit containing 2 separate capacitors. (5 pts)

c. Find the equivalent capacitance. (7 pts)

d. If one of the capacitors is filled with a dielectric material K. What is the equivalent capacitance of this new configuration? (5pts)

Problem 2. The first few Legendre Polynomials are:

$$P_0(x) = 1$$

$$P_1(x) = x$$

$$P_2(x) = \frac{3x^2 - 1}{2}$$

$$P_3(x) = \frac{5x^3 - 3x}{2}$$

Or more generally: $P_l(x) = \frac{1}{2^l l!}\left(\frac{d}{dx}\right)^l (x^2 - l)^l$

a. What is the name of the general formula given above? (3 pts)

b. Assume a spherical charge density, $\sigma(\theta) = kP_0(cos\theta)$, is glued on the surface of a spherical shell of radius R. **Using the method of separation of variables**, find the potential **outside** the surface of the sphere. (17 pts)

This problem has many parts so here are the point allocations for the necessary steps

c. Write down the potential inside and outside the sphere in terms of the Legendre polynomials and their appropriate coefficients. Explain which coefficients remain for the two cases, 1) in the limit as R approaches zero and 2) as R approaches infinity. (5 pts)

d. Determine the continuity relationships for the potential and the derivative of the potential at the boundary R. This will allow you to 1) find the relationship between the A's and B's so that you can write the boundary condition in terms of only one of the appropriate coefficients of the Legendre polynomials. (7 pts)

e. Use Fourier's "trick" to perform the integration. This will allow you to find an expression for the single remaining coefficient of the appropriate Legendre polynomial. (5 pts)

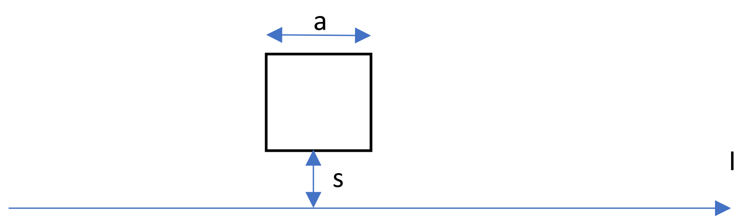
Problem 3. Starting from the flux rule (where $\mathcal{E}$ designates the induced emf). (8pts)

$$\varepsilon = -\frac{d\Phi}{dt}$$

a. Derive Faraday's law in differential form; i.e. show that

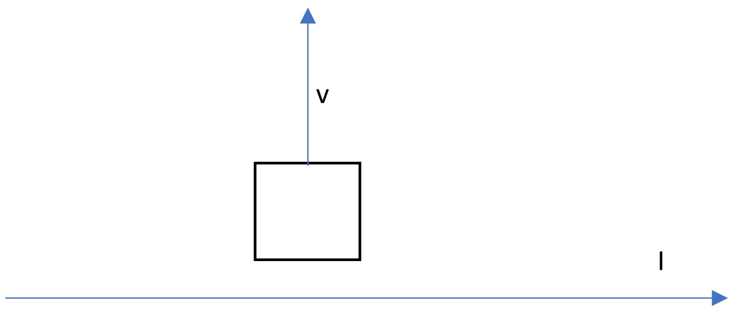
$$\boldsymbol{\nabla} X \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

b. Now consider the long straight current carrying wire below. The wire, carrying a steady current I, along with a small square loop of wire of side a lies on a flat surface a distance s from the wire. Find the flux of **B** through the loop. (6 pts)

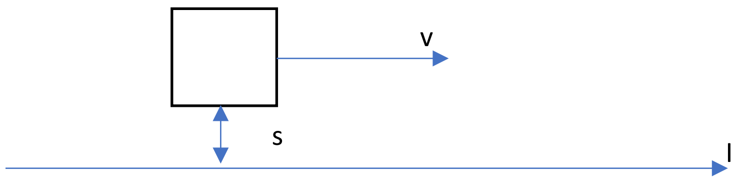


c. The value you found for the flux should not have any time dependence. Do you expect an induced emf? Explain. (2pts)

d. If the coil is now moved away from the wire at a constant velocity v, (shown below) do you expect an induced emf? Explain. (2pts)



e. If the coil is moved to the right, alongside the current carrying wire (shown below), do you expect an induced emf? Explain. (2 pts)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