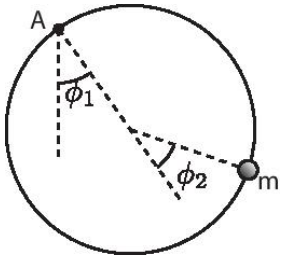
Exam

Please pick two problems to complete and indicate on the answer sheet which two you pick.

---

1. A bead of mass m is threaded on a frictionless circular wire hoop of radius R and mass m (same mass). The hoop is suspended at the point A is free to swing in its own vertical plane. (Please refer to the figure.)
   (a) Write the Lagrangian in terms of the angular position of the hoop (about A) ϕ_1 and the angular position of the bead about the center of the hoop ϕ_2.
   (b) Find the two Lagrangian equations. Simplify the equations to the case that both angles (ϕ_1 and ϕ_2) are small.
   (c) Solve the equations for the normal frequencies?
   (d) Find and describe the corresponding normal modes.



2. (a) Consider a small frictionless puck perched at the top of a fixed sphere of radius R. If the puck is given a tiny nudge so that it begins to slide down, through what vertical height will it descend before it leaves the surface of the sphere?
   (b) On a certain planet, which is perfectly spherically symmetric, the free-fall acceleration has magnitude $g = g_o$ at the North Pole and $g = \lambda g_o$ at the equator. Find $g(\theta)$, the free-fall acceleration at colatitude θ.
   (c) I am spinning a bucket of water about its vertical axis with angular velocity Ω. Show that once the water has settled in equilibrium (relative to the bucket), its surface will be a parabola. Express the height of the surface as a function of the radius.

3. Two particles whose reduced mass is μ interact via a potential energy $U = \frac{1}{k}r^2$, where r is the distance between them.
   (a) Find the "equilibrium" separation r_o, the distance at which the two particles can circle each other with constant r.
   (b) Find the frequency of small oscillations about the circular orbit if the particles are disturbed a little from the separation r_o.