UNIVERSITY OF CALIFORNIA, MERCED

Physics Graduate Group

Saturday, September 15th 2007

9:00 am – 1:00 pm

FAMILY (Last) NAME

GIVEN (First) NAME

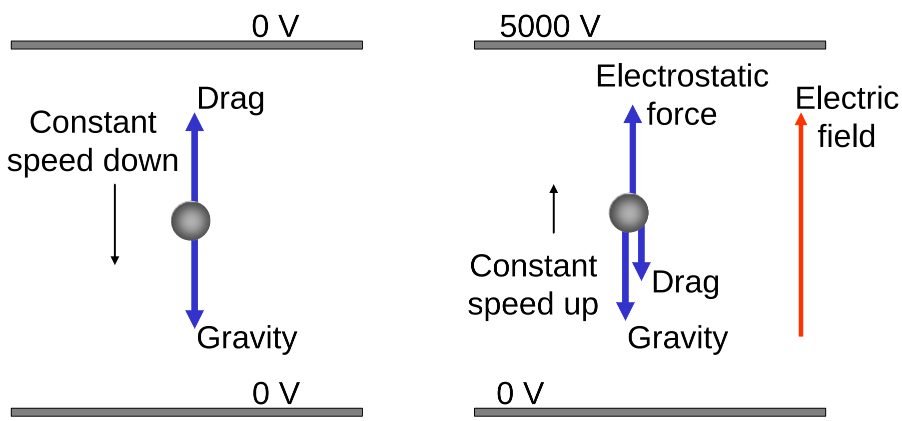
Instructions:

1 SHOW ALL WORK.
2 This is a closed book exam.
3 Do all eight (8) problems.
4 You have 4 hours to solve the problems. Exams will be collected at 1:00 pm.

Problem	Maximum	Score	Grader
1	10		
2	10		
3	10		
4	10		
5	10		
6	10		
7	10		
8	10		
TOTAL	80		

Question 1

In the Millikan oil drop experiment, which was conceived to measure the charge of an electron, small droplets of oil experience free fall under various conditions. Charged drops are introduced between two plates upon which voltages can be applied in order to create an electric field. Shown below is the case for a falling drop where there is no applied voltage, and the case where 5000V has been is applied to establish an electric field.



a) Make a list of each of the forces for the two cases depicted above, and discuss what should be the magnitude of these forces (given the mass, and charge of the droplet, and the distance between the two plates). *An example of the type of answer I am looking for is "the electrostatic force in the example on the right is given by* $F_e=NeV/D$*, where N is the number of electrons on the drop, e is the charge of an electron, V is the applied voltage, and D is the distance between the plates. This force is directed upward."*

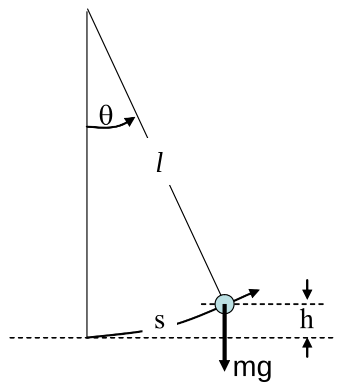
b) Make the assumption that the drop is falling at the terminal velocity for the two cases shown above, and write equations of motion (based on Newton's second law) for each of the above cases

c) Make a similar picture and free-body diagram for a third case where the applied voltage on the top plate is -5000V

d) During the experiment one measures the terminal velocity of the drop for each of the three cases (the two given to you and the one you prepared in part d). Given these velocities and the mass of the oil drop derive an expression for the total charge of the drop.

Question 2

Consider the simple pendulum consisting of a plumb bob of mass *m* swinging at the end of a light, inextensible string of length *l* (shown below). The motion is along a circular arc defined by the angle θ, as shown.



a) write an equation for the motion along the arc, *s*.

b) show that the equation of motion in (a) can be expressed as:

$$\ddot{\theta} + \frac{g}{l}\theta = 0 \qquad \text{or equivalently} \qquad \ddot{s} + \frac{g}{l}s = 0$$

c) what is the frequency of the motion?

d) what is the maximum potential energy of the pendulum assuming the maximum height is *h*?

e) Write an expression for the total energy of the system given the maximum height of *h*, and a velocity given by $\dot{s}$.

f) Show that an alternative way of expressing the potential energy is given approximately by:

$$V(s) = \frac{1}{2}\frac{mg}{l}s^2$$

Question 3

(a) The electric field generated by a perfect electric dipole p = pˆz (p > 0), centered at the origin, can be computed by taking the limit of two opposite electric charges as the separation between them goes to zero. Derive an expression for this field (magnitude and direction) at the following two points:

(i) a point in the plane perpendicular to the dipole a distance d from the origin.

(ii) a point on the positive z-axis a distance d from the origin.

(b) A capacitor consists of two parallel square plates with sides of length 1cm. It has a measured capacitance of 1 picofarad and contains no dielectric. (You may assume vacuum everywhere.) The capacitor is charged to 10 volts. Often, we only consider the electric field interior to a capacitor, but here we consider the "fringing" field that lies outside the capacitor. Compute the strength of the fringing field at a distance of 10cm from the capacitor: (i) in a direction in the plane of the plates, and (ii) in a direction perpendicular to the plates. What approximation(s) did you make?
($\varepsilon_0 = 8.85 \times 10^{-12} \mathrm{C^2/Nm^2}$)

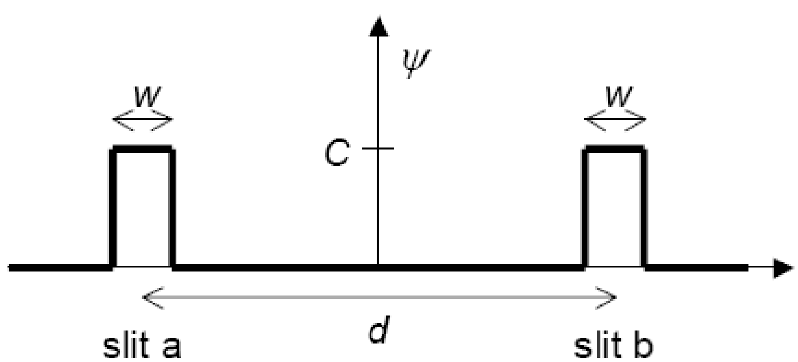
Question 4

Compute the gyromagnetic ratio (ratio of magnetic dipole moment to angular momentum) for a spinning solid sphere of radius R, with mass M and charge C. The mass and charge densities are uniform.

Question 5

The wavefunction of a particle in a double slit experiment with slit spacing *d* and slit width *w*<*d* (*w*,*d* both positive quantities) in the plane of the slits is described by

$$\psi(x) = \begin{cases} C & \text{for } -\frac{d}{2} - \frac{w}{2} \le x \le -\frac{d}{2} + \frac{w}{2} \quad \text{(slit a)} \\ C & \text{for } \frac{d}{2} - \frac{w}{2} \le x \le \frac{d}{2} + \frac{w}{2} \quad \text{(slit b)} \\ 0 & \text{otherwise} \end{cases}$$



a) Determine the normalization constant *C*.

b) Determine $\langle x \rangle$ and $\langle x^2 \rangle$ in the limit $w \ll d$, i.e. ignore any terms of order *w/d* and higher in the end result.

c) Calculate the probability density to measure a particle with momentum p between [*p*,*p*+*dp*]. Do not assume $w \ll d$.

Question 6

We would like to determine if it is possible to observe the equivalent of the Compton Effect for a target proton rather than a target electron. Calculate the maximum fractional frequency shift for an incident photon of wavelength $\lambda = 1$ Å scattering off a proton initially at rest. You may use a result known for a target electron in combination with a physical argument.

Question 7

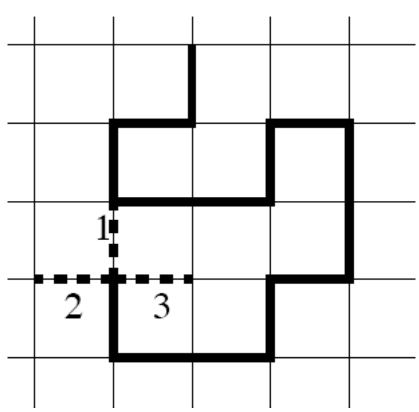
To model an elastic string like, e.g., a rubber band, we consider a linear chain of N building blocks. Each building block can be in two different states a or b. In these states the building blocks have length l_a, and l_b and energies ε_a and ε_b , respectively. The total length of the chain is $L = N_a l_a + N_b l_b$ and the total energy of the string by itself is $E_0 = N_a\varepsilon_a + N_b\varepsilon_b$ where $N_a = N - N_b$ is the number of building blocks in state a. The string is stretched by an external force f which turns the total energy of a state into $E = E - Lf$

a) Calculate the partition function of this string as a function of temperature T, the number of building blocks N , and the external force f . Introduce variables $n_i \in \{a, b\}$ that describe in which state building block i is and write the partition function as a sum over these variables n_i .

b) Calculate the average internal energy U of this string as a function of temperature T , the number of building blocks N , and the external force f .

c) Calculate the expected length of this string as a function of temperature T the number of building blocks N , and the external force f . (Hint: The expected length is a similar quantity as the expected energy. Find an expression for the expected length through a derivative similar to the derivative which we use to calculate the average internal energy.)

Question 8



A simple model for a polymer in two dimensions is that of a path on a square lattice. At every lattice point the polymer can either go straight (option 1 in the figure) or choose between the two directions in a right angle with respect to its current direction (options 2 and 3 in the figure.) Each time it bends in a right angle, it pays a bending energy ε. Thus for a given "shape" of the polymer the total bending energy of the polymer is ε times the number of right angle turns. We assume that the starting segment of the polymer is fixed somewhere on the lattice and that the polymer consists of $N + 1$ segments. Each possible shape of the polymer is a state of this statistical mechanical system.

a) How many polymer shapes have a total bending energy E where we assume $E = m\,\varepsilon$ with some integer $0 \leq m \leq N$? (Hint: First count how many ways there are to position the m right angles on the polymer of length $N + 1$ segments and then take into account that there are 2 possible choices for each right angle, namely left and right.)

b) What is the entropy $S(E, N)$ of this system? Approximate all factorials with the help of Stirling's formula.