1. **Hydrogen atom: general representation**
   An electron is bound in a hydrogen atom. We will *neglect spin* in this problem. Its state at time $t = 0$ is

$$\psi(r, \theta, \phi) = A\left(\psi_{100} + \sqrt{3}\,\psi_{211} + \sqrt{2}\,\psi_{310}\right) \tag{1}$$

   where $\psi_{n\ell m}$ are the usual normalized energy eigenfunctions of the H-atom.

   (a) Normalize $\psi(r, \theta, \phi)$

   (b) What is the expectation value of the energy in state $\psi$?

   (c) Suppose at $t = 0$ you measured the energy and observed $E_2$. Write down the resulting $\Psi(r, \theta, \phi, t)$ for $t > 0$.

   (d) A *different* electron is prepared in a state identical to the one in Equation (1). At time $t = 0$, we measure $L^2$ and find $2\hbar^2$. What was the probability of getting this value?

   (e) *After* the measurement of $L^2$ described in part (d) is made (with result $2\hbar^2$), what is the probability that a measurement of the $z$-component of angular momentum would yield $\hbar$? Explain.

Room for additional work

2. **A spin state**
   Consider an electron in the spin state

$$|\chi\rangle = A \begin{pmatrix} 1 - 2i \\ 2 \end{pmatrix} \tag{2}$$

   (a) Determine the normalization constant $A$.

   (b) What is the expectation value of $S_x$, using $\chi$?

   (c) Give the uncertainty relation (if any) for $S_x$ and $S_y$, using $\chi$.

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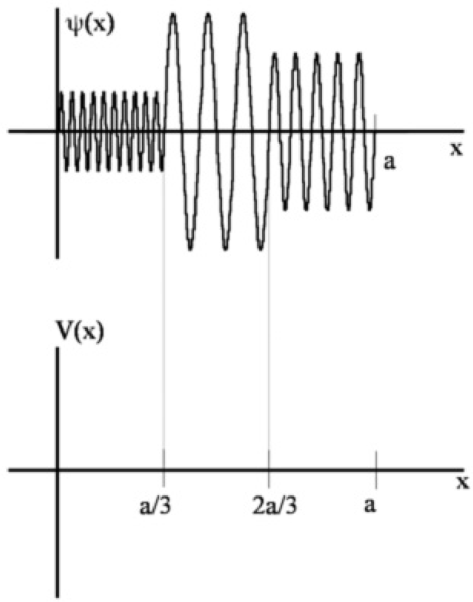
3. The real part of the wavefunction at $t = 0$ for a *highly excited energy eigenstate* of a quantum mechanical system is shown at the right. (The wave function is zero for $x < 0$ and $x > a$).
   (a) In the empty graph below, sketch a potential energy $V(x)$ and total energy, that are consistent with this wavefunction. Clearly show relative lengths on your vertical scale and, *clearly explain the reasoning you used to draw your graph*.
   (b) Given the description of the wavefunction, is $\langle x \rangle$ greater than, less than or equal to $a/2$? Explain.
   (c) Will this wavefunction change with time, or not? Explain.
   (d) Will the probability density associated with this wavefunction change with time, or not? Explain.
   (e) Will $\langle x \rangle$ change with time, or not? Explain.
   (f) Is the uncertainty in the momentum of this particle $\sigma_p$, zero? If yes, explain why. If not, what bounds can you set for it?



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