2016 Fall Graudate Prelim Exam: Quantum Mechanics

Please pick two (2) problems to complete. Please indicate which two you pick on the answer sheet.

1. (a) Explain what is wrong with the classical model of atom, where electrons orbiting the nucleus like the earth orbiting the sun.
(b) The element Bismuth-210 has a half-life of 5 days. A piece of pure bismuth-210 weights 4 grams initially. How much bismuth-210 will remain 20 days later?
(c) There is only one bismuth-210 atom initially. 5 days later how much bismuth-210 will remain?
(d) By shining photons with wavelength L on a metallic surface, no photoemission is produced. Which way can produce photoemission from the surface? (A) Increase the laser intensity by sending in more photons per second. (B) Increase the wavelength. (C) Increase the frequency. Explain why.
(e) Estimate the lowest possible energy of a quantum particle with mass m in a one-dimensional box of length L by using the uncertainty relation $\Delta x \cdot \Delta p = \hbar/2$. Here $\hbar = h/(2\pi)$.
(f) Estimate the lowest possible energy of a quantum particle with mass m in a one-dimensional box of length L by using de Broglie's matter wave theory $p = h/\lambda$.
(g) A system has 4 eigenstates $\psi_{1,} \psi_{2,} \psi_{3,} \psi_4$ with corresponding eigen-energy $E_1 < E_2 < E_3 < E_4$. A state is initially prepared as $\psi = a_1 \psi_1 + a_2 \psi_2 + a_3 \psi_3 + a_4 \psi_4$, where the coefficients are complex numbers. A measurement reveals the energy of the state as E_3 . What is the probability of seeing the state in ψ_1 ? What value will you obtain when you make a second measurement of energy?

2. The spin of an electron can be represented as a two component spinor. Let's take the z-direction spinors as the basis: $\chi_\uparrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \chi_\downarrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.
(a) A spinor is given by $\chi_1 = a \cdot \chi_\uparrow + b \cdot \chi_\downarrow$, where *a* and *b* are complex numbers. If it is well-defined, what is the relation between *a* and *b*?
(b) What is the probability of seeing χ_1 pointing downward in the z-direction?
(c) An apparatus is modeled by the operator $G = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, what state will be produced after the spinor χ_1 passes the apparatus?
(d) The spinor pointing upward in the x-direction is $\chi_{x\uparrow} = (\chi_\uparrow - \chi_\downarrow)/\sqrt{2}$, the spinor pointing downward in the x-direction is $\chi_{x\downarrow} = (\chi_\uparrow + \chi_\downarrow)/\sqrt{2}$, and the spin operator along the x-axis is $S_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. What is the probability of seeing χ_1 pointing upward in the x-direction?
(e) A Hamiltonian *H* has the properties $H \chi_\uparrow = E_1 \chi_\uparrow, H \chi_\downarrow = E_2 \chi_\downarrow, E_1 < E_2$. Given the time-dependent Schrodinger equation $i\hbar \partial \chi = H \chi$ for a state χ and the initial state being χ_1 , how does the state look like at time *T*?

3. The ground state of the electron in a hydrogen atom is $\psi_0(r) = A \cdot e^{-r/a_0}$, where a_0 is the Bohr radius.
(a) What is the probability density of finding the electron at distance $10 a_0$ from the nucleus?
(b) Please determine *A*.
(c) Please find the expectation value of the operator *r (, that is* $\langle r \rangle$ *,)* of the state ψ_0 .
(d) Prove (formally) that, no matter how hard you try to construct a state from superposition, its energy expectation value cannot be smaller than the ground state energy.