Prelim : Quantum Mechanics
Fall 2013
Saturday September 14 2013
Choose 2 out of 3 questions

Problem 1: Hydrogen Atom (20 points)

An electron is bound in a hydrogen atom. Its state at $t = 0$ is

$$\psi(r,\theta,\varphi) = \frac{1}{\sqrt{6}}\left(\psi_{100} + \sqrt{2}\psi_{211} + \sqrt{3}\psi_{310}\right) \qquad (1)$$

where ψ_{nlm} are the usual normalized energy eigenfunctions of the H-atom. We will *neglect spin* in this problem.

a. What is the expectation value of the energy in state ψ ?

b. Suppose at $t = 0$ you measured the energy and observed E_2 (i.e. $n = 2$). Write down the resulting $\psi(r,\theta,\varphi,t)$ for $t > 0$.

c. *Another* atom is prepared in the original state given by equation (1). At $t = 0$ we measure the angular momentum squared and find $2\hbar^2$ (i.e. $l = 1$). What was the probability of getting this value?

d. *After* the measurement of L^2 described in part (c) is made (with result $2\hbar^2$), what is the probability that a measurement of the z-component of angular momentum would yield $\hbar$? Explain.

Problem 2: Harmonic Oscillator (20 points)

Several relations using raising and lowering operators for the quantum Harmonic Oscillator are reproduced below:

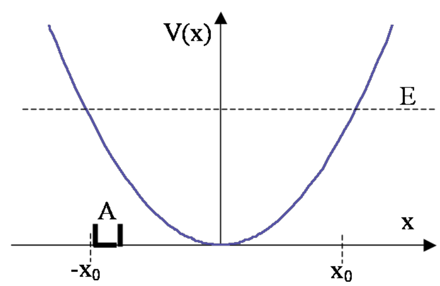
$$a_\pm = \frac{1}{\sqrt{2\hbar m\omega}}(\mp i\hat{p} + m\omega x), \qquad a_+\psi_n = \sqrt{n+1}\ \psi_{n+1}, \qquad a_-\psi_n = \sqrt{n}\ \psi_{n-1}$$

And, $$\int_{-\infty}^{+\infty} f^*(a_\pm g)\ dx = \int_{-\infty}^{+\infty} (a_\mp f)^* g\ dx$$

a. Using the above, show that the eigenvectors of the Hamiltonian oscillator are orthogonal, i.e. prove $\int_{-\infty}^{+\infty} \psi_n^*(x)\psi_m(x)\ dx = 0$, if n is not equal to m.

   Hint! Try evaluating $\int_{-\infty}^{+\infty} \psi_n^*(x)a_+a_-\psi_m(x)\ dx$

b. Consider a particle in a quantum mechanical harmonic oscillator, with energy E (shown in the figure). Consider two possibilities that could be true for this figure:



   - Possibility #1: this particle is in its ground state.
   - Possibility #2: this particle is in a highly excited state, $n \gg 0$.

   For which of these two possibilities is the particle more likely to be found within the region $-x_o \le x \le -x_o + A$ (marked "A" in the figure), or is it equally likely in either case? Briefly explain.

Problem 3: Commutators & Uncertainty Principles (20 points)

Prove the famous "(your name) uncertainty principle," relating the uncertainty in position $(A = x)$ to the uncertainty in energy $(B = p^2/2m + V)$

$$\sigma_x \sigma_H \geq \frac{\hbar}{2m} | < p > | .$$

For stationary states, this doesn't tell you much—why not?