Physics Preliminary Exam Spring 2010
Paper 2 – Classical Mechanics
Feb 17th 2010
6-8pm

Attempt 2 out of the 3 problems

Problem 1

A cylinder of mass M, radius R and height H is suspended from a spring with spring constant k. The cylinder is then partially submerged in water of density ρ. At equilibrium the cylinder sinks to half its height. At t=0, the cylinder is submerged to 2/3 of its height and then released.

a. Find and solve the equation of motion of the centre of mass of the cylinder.

b. If we now take into account a resistive force proportional to the velocity of the cylinder coming from the “drag” due to the water, what is the new equation of motion?

Problem 2

a) Give a real life example of a system that will behave as a weakly damped harmonic oscillator.

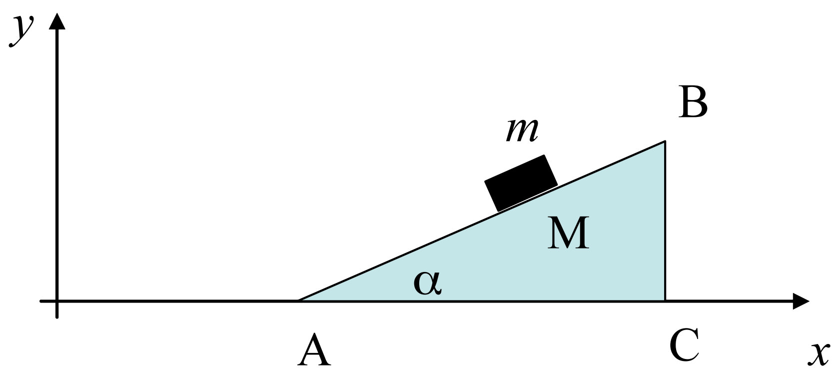
b) Set up the problem and write an equation of motion.

c) Give the solution and describe the meaning of each of the terms in the solution.

d) Make a sketch of the position as a function of time where the key physical concepts are labeled.

Problem 3

Consider the system in the figure below. A wedge with projection ABC on the xy plane of mass M is able to slide freely on the horizontal plane. The perfectly smooth slant surface AB of the wedge makes the angle α with the horizontal. On this surface a particle of mass *m* is free to move in the xy plane. The problem is to determine the motion of *m* as well as the wedge.



a) Describe the approach to solving this problem using the Lagrangian formalism.

b) Show that it is possible to write *L* as

$$L = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m\dot{y}^2 + \frac{1}{2}m\left(\dot{x} + \frac{\dot{y}}{\tan\alpha}\right)^2 - mgy$$

c) Show also that

$$M\ddot{x} + m\ddot{x} + \frac{m\ddot{y}}{\tan\alpha} = 0$$

$$m\ddot{y} + \frac{m\ddot{x}}{\tan\alpha} + \frac{m\ddot{y}}{\tan^2\alpha} + mg = 0$$

d) What is interesting about the case where m/M → 0?

Physics Preliminary Exam Spring 2010
Paper 2 – Quantum Mechanics
Feb 18th 2010
6-8pm

Attempt 2 out of the 3 problems

Problem 1

Non-degenerate perturbation theory:

Consider a three-level system with the initial Hamiltonian $H_0 = \begin{pmatrix} e_0 & 0 & 0 \\ 0 & 2e_0 & 0 \\ 0 & 0 & 3e_0 \end{pmatrix}$.

A perturbation is applied to this system with the form $H_1 = \begin{pmatrix} \lambda_1 & 0 & \theta_1 \\ 0 & \lambda_2 & \theta_2 \\ \theta_1 & \theta_2 & 0 \end{pmatrix}$,

where $\lambda_{1,2}, \theta_{1,2}$ are small perturbations when compared with e_0. Please answer the following questions:

a) What are the first order corrections to the energies of the eigenstates?

b) What are first order corrections to the ground state wave vector?

Problem 2

Consider an infinite square well with the length L and the potential barrier:

$$V_i(x) = \begin{cases} 0, & 0 < x < L \\ \infty, & otherwise \end{cases}$$

The eigenstates and eigenenergies are respectively:

$$\psi_n = \sqrt{\frac{2}{L}} \sin(\frac{n\pi}{L} x) \text{ and } E_n = (\frac{n^2\pi^2}{2mL^2} \hbar^2)$$

Initially the particle is in the ground state of $V_i(x)$.

Now, the barrier is SUDDENLY switched to 2L and the potential becomes:

$$V_f(x) = \begin{cases} 0, & 0 < x < 2L \\ \infty, & otherwise \end{cases}.$$

Please answer the following questions:

(a) what are the new eigenstates and eigenenergies after the sudden switch?

(b) express the initial state (ground state of the initial potential) in the new basis.

Problem 3

(a) Derive the expression for the normalized wavefunctions $\Psi_n(x)$ for a particle in a one-dimensional infinite square well, where the left wall is at x = −L/2 and the right wall is at x = L/2, i.e. the potential is:

$$V(x) = \begin{cases} 0 & |x| < L/2 \\ \infty & |x| > L/2 \end{cases}$$

Here n denotes the principal quantum number. Be sure to state the allowed values of n.

b) What are the corresponding eigenenergies, E_n for each value of the quantum number? (Denote the particle mass by m.)

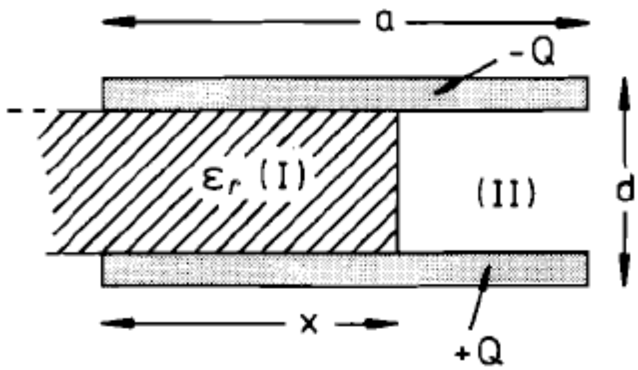
c) Define the parity operator P acting on a generic wavefunction $\Psi(x)$ by $P\Psi(x) = \Psi(-x)$. What is the eigenvalue of P for each state Ψ_n? (It might help to sketch the first few wavefunctions).

d) Which transitions are dipole forbidden according to the parity selection rule? That is, using the results from part (iii), determine the values of n and n′ for which the matrix element $\langle \Psi_n | x | \Psi_{n'} \rangle$ vanishes. You shouldn't actually work out any explicit integrals here. Simply use symmetry arguments.

Physics Preliminary Exam Spring 2010
Paper 3 – Electromagnetism
Feb 19th 2010
6-8pm

Attempt 2 out of the 3 problems

Problem 1



A capacitor (area $F=a\bullet b$, separation d) is partially filled with some dielectric material with a dielectric constant $\varepsilon_r>1$ (see figure). The remaining space between the capacitor plates is empty. The bottom plate carries a charge Q and the top plate carries a charge $-Q$. Any stray fields can be neglected.

a) What is the relationship between the electric field **E** and the dielectric displacement **D** in the two regions, (I) and (II), between the capacitor plates?

b) What can be said about D_I/D_{II} and E_I/E_{II}?

c) What is the relationship between D_I, D_{II} and the surface charge densities σ_I, σ_{II}?

d) Calculate the **E** and the **D** field for the entire space between the capacitor plates.

e) Compute the electrostatic energy density.

f) By considering the energy change due to a displacement of the dielectric material by a distance dx determine the force acting on the dielectric material.

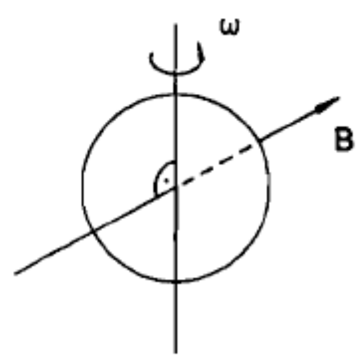
Problem 2

Find the electrostatic potential and electric field in the following cases:

a. A point charge q is placed at a distance d above a conducting plane that is grounded. Find the potential and field everywhere above the plane.

b. A disc of radius R has a hole of radius a cut into it. The center of the hole coincides with the center of the disc. The disc has a charge density σ. Find the potential and field along the axis of the disc. Discuss your results for limiting cases defined by the values of R and a.

Problem 3



A circular ring with radius, R, rotates with a constant angular frequency, ω, about its diameter (see Figure). A homogeneous magnetic induction, **B**, is oriented perpendicular to the rotation axis of the ring.

a) Calculate the time-dependent induced voltage in the ring.

b) The ring is made of a metal wire with conductivity, σ. What current, $I(t)$, exists in the ring, if one assumes that the current is distributed evenly over the cross-section of the wire?